ESSENTIALS OF LOGIC

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ESSENTIALS

OF

LOGIC

BY

A. WOLF, M.A., D.Lit.

Professor of Logic and Scientific Method in the University of London;
Fellow of the University of London, University College;
Formerly Fellow of St. John's College, Cambridge;
Author of "Essentials of Scientific Method,"
"Studies in Logic," etc.

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W.C.

PREFACE

THIS little book is a companion to the volume on *Essentials of Scientific Method* published in 1925. The two volumes between them present the essentials of Logic and Scientific Method as clearly and simply as possible. The aim throughout is to explain all that is most valuable in the traditional account, to omit what is inaccurate or irrelevant, to make additions or modifications where necessary, and to develop the whole subject logically. Above all there is a steadfast avoidance of discussions which only confuse the beginner, and an endeavour to present the subject in a straightforward, positive manner. Time enough for criticisms and controversies when the essentials have been mastered!

It cannot be emphasized too strongly that Logic and Scientific Method cannot be learned properly by mere reading, any more than Algebra and Geometry can. The learner must get sufficient practice in the analysis of actual arguments and investigations. Adequate material for such practice is provided in the author's *Exercises in Logic and Scientific Method,* and sufficient help with them will be found in the *Key to the Exercises.*

As already stated in the Preface to *Essentials of Scientific Method,* the philosophical problems arising out of the study of Logic and Scientific Method will

be discussed in another volume to be published in the near future.

I wish to express my gratification with the warm welcome given to the *Essentials of Scientific Method*, and the hope that its present companion volume may be found equally helpful.

A. WOLF.

UNIVERSITY OF LONDON,
September 1926.

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ESSENTIALS OF LOGIC

CHAPTER I

INTRODUCTORY

§ 1. *The Scope of Logic.*

Logic is the study of the general conditions of valid inference (or of proof). To make this description intelligible it is necessary to explain some of its constituent terms, more especially the terms *inference* and *valid.*

Inference. An inference is an inferred judgment, that is, a judgment derived from another judgment, or from other judgments. All knowledge and all beliefs consist of judgments. In ordinary usage it is customary to distinguish between *knowledge* and *belief.* There are some things which we claim to *know,* there are others which we do not claim to know, but which we still *believe.* When used correctly the term *belief* is the more general term of the two, and includes *knowledge.* If we *believe* what we do *not* claim to *know,* we certainly believe what we *do* claim to know. In this sense, *knowledge* may be described as adequately justified belief, whereas beliefs not adequately justified may be described as *mere beliefs.* Used in the wide sense just explained, namely as including both *knowledge* and *mere beliefs,* the term

belief is synonymous with *judgment,* as the term is used in Logic and Psychology. Now broadly speaking there are two kinds of judgments in respect of their origin, or felt origin. Some are obviously derived from other judgments, while others are not so derived. For example, I look at the sky at sunset and, seeing that it is red, I believe that it will be fine to-morrow. My belief that it will be fine to-morrow is derived from (1) my observation of the red sky, and (2) my belief in the connection between a red sky at sunset and fine weather to follow. Such a derived belief or judgment is called an *inference.* On the other hand, my belief that the sky is red is not derived, but is the result of direct observation, while the belief in the connection between a red sky at sunset and fine weather to follow, may be either the immediate result of suggestion, or an inference derived from my belief in the credibility of my informant, or from my knowledge of the physical facts involved. Inferences, then, are *derived judgments*; judgments which are not derived from other judgments may be called *immediate or intuitive judgments.* Such immediate or intuitive judgments result either from perception by means of the senses (judgments of perception, or *perceptual judgments*), or from that kind of intellectual intuition to which we owe such self-evident truths as the axioms of geometry, etc. (*intuitive judgments* in the stricter sense). It is not always easy to distinguish an immediate from an inferential judgment. With the progress of knowledge and critical discrimination it is easily recognized that many judgments commonly regarded as immediate are really inferential. Still, the difficulty should not be exaggerated. The main point with which we are concerned at the moment is,

that Logic, unlike Epistemology (or the Theory of Knowledge), is not concerned with all kinds of judgments, but only with those which are professedly derived from, or based upon, other judgments. Logic is the study of *inferences* not of beliefs generally.

A word may be added here about the relation of *inference* to *proof,* which terms appear almost as synonyms in the definition of Logic given above. In every argument there are two things: (1) the premises (or data, or evidence) and (2) the conclusion (or the inference). The conclusion is *inferred from* the evidence, and, if it is inferred accurately, the evidence is said to *prove* the conclusion. Thus *inference* and *proof* are simply different aspects of the same thing.

Validity. It was stated above that Logic is concerned primarily with *valid* inference. The meaning of this should be fairly clear. "Valid" means the same as "correct," or "accurate," or "sound"; and at some time or other everybody distinguishes between (usually his own) "correct" inferences and (his opponent's) "incorrect" inferences. Still, there is at least one point which calls for careful consideration. A *valid* inference is not the same thing as a *true* inference. Careful thinkers usually endeavour to make their inferences both valid and true; but it is possible for inferences to be valid without being true, or to be true without being valid. An inference is *valid* when it is reasonably justified by the evidence adduced in support of it (that is, by the judgments from which it is derived); it is *true* if it is in accord with the relevant facts, that is, if it describes the facts concerned approximately as they are. Now gamblers and others sometimes make rash inferences which

turn out to be true, though they were not really valid. On the other hand, judgments are sometimes disproved by a process known as that of reduction to absurdity (*reductio ad absurdum*), that is, by drawing from them valid inferences which are absurd, that is, inferences which are valid but obviously not true. Now Logic is only concerned with the study of *valid* inference. This does not mean that Logic disregards truth. Far from it. Logic is certainly concerned with the formulation of the *true* conditions of valid inference. It is sheer necessity that compels Logic, as it compels all the sciences, to confine itself to a limited set of problems. The study of the conditions of *valid* inference involves the study of the relations between inferences and premises (that is, the judgments from which the inferences are derived), and that is a sufficiently important task by itself. The study of the conditions of *true* inference would involve in addition the study of the truth of all possible premises—an obviously impossible task, and utterly opposed to the division of labour to which science, as well as industry, owes its advancement. The careful thinker will naturally see to it that his data, or premises, are true before he draws any conclusions from them. But for the study of inference as such it is necessary to isolate the problem of validity—to study the main types of inference, and the relations between the inferences and the premises when the inferences may be said to be justified by the premises. This involves abstraction from the truth of the premises. But then every science abstracts from something in order to simplify its problems sufficiently to make them manageable. To abstract from anything, however, is not the same thing as to reject or to ignore it utterly; it is simply

not to deal with it at the same time as certain other problems are being dealt with, with the clear understanding that the neglected problems or aspects must receive adequate attention as soon as they become relevant in any actual, complex situation.

Generality. Logic is concerned with the *general* conditions of valid inference. Every actual argument relates to some *particular* problem mathematical or astronomical, physical or chemical, political or economic, legal or moral, etc. The full consideration of an argument must consequently take into account its special subject-matter, and requires a knowledge of it. If Logic were to attempt to deal with arguments in all their detail, then (as was already suggested in another connection) it would have to absorb all the sciences, etc.—an obviously impossible and absurd enterprise. Logic, accordingly, abstracts from the special subject-matter of each argument, and confines itself to the study of the main *types*, or *kinds*, of argument, and the *general* conditions of validity concerning each type. This kind of procedure is not peculiar to Logic. Every science is like it in this respect. Like Nature herself, every science is careful of the type, not of the individual, except as a specimen of the type. At the same time Logic, like Mathematics, is more general or abstract than most sciences, and this characteristic generality of Logic, as of Mathematics, finds expression in the extensive use of symbols. Symbols in Logic stand mostly for the subject-matter to which the argument relates, but they are a device for abstracting from the peculiarities of the various kinds of subject-matter, and so help one to concentrate on the *general* character of the various types of argument, and the *general* conditions of their validity.

§ 2. *The Formalism of Logic.*

The fact that Logic is interested in the *validity* of inference, and only in the *general* conditions of such validity, may also be expressed by saying that Logic is interested in the *general relationships* between inferences and the premises from which they are derived, or by which they are justified. Such general relationships, in which the peculiarities of their terms (or subject-matter) are abstracted from, may be described as "forms," or "forms of argument"—the actual terms (or subject-matter) constituting the "matter" of the argument. Logic may therefore be said to be concerned with "forms" of argument. Hence the occasional reference to the "formalism" of Logic. It means, however, no more than that Logic is concerned with *types* of argument, and with the *general* conditions of their validity. It means no more than that Logic is *general* and rather *abstract* in its methods and aims. In this respect, however, the difference between Logic and the Sciences is at most one of degree, not one of kind.

§ 3. *The Function of Logic.*

The main function of Logic is to make intelligible, or to explain, the general nature of valid inference; not to enable one to argue or to reason more correctly, though it may do that also incidentally. The main purpose of every science is to enable people to *understand* things, not to *do*, or to make, them. Astronomy does not profess to construct stars or stellar systems, or to teach the stars their courses; its aim is to describe and to explain the stars and their movements. People do not wait for physiology to teach them to eat and

drink, to walk and run, etc. If people could not do these things without the aid of physiology, physiology itself could never come into existence. Similarly, people can and do reason correctly without the aid of Logic; if they could not do so, Logic itself would not exist. God (as Locke remarked) has not been so sparing to men as to make them barely two-legged creatures, leaving it to Logic to make them rational. Rather it is the native rationality of man that has made Logic itself possible. The main aim of Logic is not to teach people to reason correctly, but to explain what happens when they do reason correctly, and why some reasoning is not correct. At the same time just as a knowledge of the sciences is generally useful in some way or other, although such utility is not their main aim, so a knowledge of Logic may be, and should be, useful in checking one's conclusions, if one is not so desperately self-complacent as to be beyond all help and all improvement.

CHAPTER II

JUDGMENT AND TERMS

§ 1. *Judgment and Proposition.*

A judgment or belief when expressed in language is commonly called a proposition. Even our private thoughts, that is, even the judgments which we do not at the time communicate to others, or put on record, are carried on largely through the medium of inarticulate language. But it is obvious that judgments cannot be explained to others, or discussed with them, except through the medium of propositions. For this reason we shall be concerned mainly with propositions, and the relations which must exist between propositions in order that one proposition (called the inferred proposition, or simply the inference, or the conclusion) may be said to be legitimately derived from another or others (called the premises, or the evidence, or the data). It should be noted, however, that the term *proposition* is used in a somewhat more extended sense than has been indicated so far. It denotes not only the verbal expression of an *actual* judgment or belief, but also the verbal expression of a suggestion, or supposition, or a merely *potential* judgment. Among thoughtful, critical people suggestions and suppositions *as such* play an important rôle. Only the uncritical and conceited dogmatist regards whatever he takes into his head as an indis-

putable intuition, if not as a divine inspiration. The critical person, who alone has the making of a man of science, turns such thoughts round and round, treats them as mere suggestions, or suppositions, or "propositions," and scrutinizes them with caution in the light of the available evidence. All such suggestions, when expressed in language, are also called propositions. It is, moreover, a familiar fact that what one person firmly believes, another may as firmly disbelieve, while yet a third may regard it as a suggestion worth considering. It will be most convenient, accordingly, to use the expression *proposition* for the expressed content of any thought such as may be believed, or disbelieved, or merely understood and considered. In other words our use of the term proposition makes abstraction from the element or moment of belief, and any statement which can be true or false is called a proposition, no matter if it is believed, disbelieved, or merely under consideration.

§ 2. *Implication and Inference.*

It should be fairly obvious that one proposition can be inferred legitimately from another proposition, or from other propositions, only when the other proposition or propositions *imply* it. In fact *implication* and *inferribility* are correlative terms—to say that certain conclusions are *inferrible* from certain premises, is equivalent to saying that those premises (either separately or jointly) *imply* those conclusions. The problem of inferribility is therefore the same as that of implication. And the question of the general conditions of *valid inference* may be answered by considering the *implications* of the different types of propositions regarded as potential premises. It is

clear from ordinary usage that when we distinguish between the *meaning* of a statement and its *implication* we really distinguish between its more obvious and its less obvious sense. That, at all events, is the way in which we usually distinguish between what a person *says* and what he *implies* (or insinuates, etc.). The difference between *meaning* and *implication* is only a difference of degree at most, and it is not always easy to determine at what point precisely the (direct) *meaning* of a statement ends, and its (indirect) *implication* begins. For the purpose of the logical problem the distinction is of no fundamental importance, and the term *implication* may be used in an inclusive sense, covering the more obvious, direct meaning of statements as well as their less obvious, or indirect, meanings.

We may proceed now to the consideration of the main types of proposition and their chief implications.

§ 3. *Judgment as Intellectual Orientation.*

Life is primarily a process of continuous orientation; it has to find its bearings among its surroundings, and struggle for its survival by employing suitable responses, or re-actions, to its environment. At the lower planes of life this orientation is more or less blind, instinctive or impulsive, though not entirely blind, and certainly not merely mechanical. The method of trial and error, the desperate efforts by which living organisms of all kinds fight for life, frequently results in improved adaptation. In some way experience teaches even the lowliest organisms. At the human stage of life, however, there emerges a more clear-sighted form of orientation, alongside of the more primitive form. The new form of orienta-

tion is intellectual in character, and consists of all that we call knowledge, belief, or judgment. Under specially favourable conditions knowledge may be, and is sometimes, pursued for its own sake. But, in the first instance, it is simply an extension of the more primitive type of orientation; it is an intellectual instrument in the struggle for existence. Considered biologically the chief feature of our intellectual orientation is an enormous improvement in the extent to which we learn from past experience, not only from our own individual experience, but also from the experience of others. Learning from experience is by no means new at the human level—even young chicks learn from experience. But at the human level the process sometimes becomes clear-sighted and articulate. Objects and situations which we have already experienced, and which we have learned how to deal with, are retained in ideas, or concepts, which are helpful in new situations of the same kind. Again, even lower animals, certainly chimpanzees, are taught by experience to apprehend vaguely that certain things are connected with one another, or are dependent on one another—say, the falling of bananas out of a certain basket with the pulling of a string attached to the basket. At the human stage, however, such apprehension becomes clearer, and is articulated in general ideas of laws of interdependence, etc., which can be applied to real or imaginary situations with a foresight of the probable result. The biological advantages of such purely intellectual or imaginary experiments, as distinguished from actual trial-and-error experiments, are obvious in all cases in which the actual experiment might prove injurious or fatal. In course of time intellectual interests

extend beyond problems of immediate practical importance, and, along with increasing remoteness from the practical needs of the moment, there goes an increase in complexity. The highest intellectual efforts are directed to the disinterested study of the order and connections in the phenomena of nature; though there is always the hope that, in one way or another, even the most disinterested knowledge may contribute towards the amelioration of human destiny. But what concerns us most now is to note that all this intellectual work consists of judgments of various kinds, interlinked in various ways, and that the increasing complexity of intellectual effort consists in the derivation of some judgments from others, which is the essence of reasoning, or drawing inferences.

§ 4. *Subject and Predicate.*

In the simplest kinds of judgments what happens is this: something confronting us, and in some way of interest to us, is recognized as being an object, or quality, etc., of a certain kind with which we are already familiar from previous experience, and of which we accordingly have a concept (or idea); or some concept which first suggested itself is denied of the object in question, usually because some other concept seems to fit better. Such judgments when expressed in language may assume the form of such simple utterances as "Rain," "Fog," "A rainbow," "Cold," "Dark," "Not foggy," etc. What is expressed, in each of these cases, is the concept under which the observed object, etc., is brought, or by which it is apprehended, or interpreted. The concept by means of which the object, or situation, etc., is recognized, or interpreted, is called the *predicate*; the

observed object, situation, etc., which called for recognition, or interpretation, is called the *subject.* In the simplest cases, like the above, the subject is not expressed in language at all; but when it is expressed in language (by means of a pronoun, a noun, or nominal phrase) the verbal expression is also called the subject. In somewhat more developed judgments the object, or situation, etc., requiring further elucidation is not apprehended so vaguely as to be inarticulate except for the predicate, but is at once apprehended under one concept, though still requiring further elucidation with the aid of other or more determinate concepts. In such cases the subject, as well as the predicate, is expressed in language. Subjects and predicates (also their verbal expressions) are called the *terms* of the judgment (or of the proposition). And we are so accustomed to propositions with at least two terms (subject and predicate) that judgments which would more naturally be expressed by means of the predicate only, have a dummy subject added to them. Hence such impersonal propositions as "It is foggy," or "It is cold," etc., instead of merely "foggy," "cold," etc. The *judgment* itself, of course, never involves less than two terms; only in the simplest cases, the subject is really inexpressible in language, because it is apprehended too vaguely, and so its *verbal expression* is more natural if without an expressed subject.

Judgments and propositions are of varying degrees of complexity. The more complex ones involve three, four, or even more terms. In fact, the more complex propositions are best treated as composed of simpler propositions, related in certain ways, just as the simpler propositions are composed of terms. But it is important to realize from the first that intellectual

activities are essentially continuous, not discrete. A judgment is not produced by putting together two discrete terms, nor are more complex judgments and inferences produced by putting together several discrete judgments. The whole process is much more complex than that; it is more like a continuous growth in which the comparatively simpler objects pass into more and more complicated and more differentiated wholes by the assimilation of new materials. It is important to bear all this in mind, as an erroneous view is easily encouraged by our pre-occupation with propositions, which in a sense really are put together from discrete words or letters. In our actual mental experience what happens is quite different from such mere addition or juxtaposition of separate units. There is always a continuum—consisting of a comparatively vague background (or hinterland) out of which some objects only just emerge more definitely, while others are in the focus of consciousness, and are most clear and distinct of all. These are psychological matters with which we are not directly concerned; but it is desirable to avoid misapprehensions which may easily distort one's conceptions of topics which are of more immediate logical interest.

We may proceed now to the consideration of the principal types of propositions and their implications.

CHAPTER III

CATEGORICAL PROPOSITIONS AND THEIR IMPLICATIONS

§ 1. *The General Character of Categorical Propositions.*

In the simpler, and commonest, type of proposition a predicate is simple affirmed or denied of a subject. By "simply" is here meant unconditionally, or without reservation, and without implying any necessary connection between the terms of the proposition. Such propositions are called *categorical* propositions. The following may serve as examples of such propositions. *The earth is a planet. The earth is not flat. All planets move in elliptical orbits. No planets are fixed. Some stars are self-luminous. Some planets are not self-luminous.* If we abstract from the special character of the terms, and the quantity of the subject (that is whether one, some, or all are referred to) then the general character of categorical propositions may be represented by the formulæ *S is P* and *S is not P*, where *S* stands for any *subject*, and *P* for any relevant *predicate*. These and similar symbolic formulæ are usually called *forms*, because they abstract from the subject-matter, or actual terms, of propositions—that being the only way in which *types* of propositions can be studied. But it should be noted once for all that it is always assumed that actual propositions have definite terms, even if their special character is not at

the moment under consideration. There is no such thing as a proposition which has form but no subject-matter, any more than there is such a thing as a proposition which has subject-matter and no form. But the forms of propositions may be studied to a considerable extent apart from their actual terms, and terms may be studied to some extent apart from the propositional forms in which they occur.

§ 2. *The Quality of Categorical Propositions.*

It will be observed that in the above examples of categorical propositions the predicate is *affirmed* of the subject in some cases, and *denied* in others. The difference between *affirmation* and *negation* is called a difference of *quality*, and it is the only difference in respect of quality. Every proposition is either *affirmative* or *negative*, according as the predicate is affirmed or denied of the subject. Thus, to revert to our previous examples, *The earth is a planet, Some stars are self-luminous, All planets move in elliptical orbits*, are all *affirmative* propositions; on the other hand, *The earth is not flat, Some planets are not self-luminous, No planets are fixed*, are all *negative* propositions.

The relation between affirmative and negative propositions calls for some consideration. On the one hand, it is true, in a very real sense, that the difference between affirmation and denial is fundamental; it cannot be effaced by reducing either to the other. Given an unambiguous *subject* and an unambiguous *predicate*, say S and P, then there are three possibilities. One may affirm P of S, and the result will be an affirmative proposition, *S is P*. Or one may deny P of S, and the result will be a negative proposition *S is not P*,

Lastly, one may not know whether *P* should be affirmed or denied of *S*; but in this case the result is, for the time being, only a problem, *Is S P*?—a question, not a proposition. Now of the two possible propositions, *S is P*, *S is not P* (*S* and *P* being unambiguous in every way), the second is really a rejection of the former. It comes to the same thing whether one says, "I disbelieve *S is P*," or whether he says, "I believe *S is not P*." Now *belief* and *disbelief*, it should be obvious, are fundamentally different and incompatible attitudes towards the same suggestion. To that extent, *affirmation* and *negation* are fundamentally different from each other.

In some respects, however, the difference between affirmative and negative propositions is not one of fundamental importance, but rather one of convenience. Namely, what is essentially the same suggestion, or belief, may be expressed either in an affirmative proposition, or in a negative proposition, according to circumstances. But, of course, the *terms* will have to be different to some extent. For instance, the negative proposition *No planets are fixed* means the same thing as the affirmative proposition *All planets move*; similarly the affirmative proposition *Air is transparent* means the same thing as the negative proposition *Air is not opaque*; the negative proposition *No railway tickets are transferable* means the same as the affirmative proposition *All railway tickets are non-transferable*; and so on. Or, to take an example already used in another connection, the negative statement, "The proposition *S is P* is not true" means the same thing as the affirmative statement, "The proposition *S is not P* is true"—a very different thing from maintaining both that *S is P* and that *S is not P*, which (as has

already been explained above) would be impossible, so long as *S* and *P* are quite unambiguous.

§ 3. *The Quantity of Categorical Propositions.*

The examples of categorical propositions already given will have shown that they vary not only in the way in which the predicate is asserted of the subject, but also as regards the extent of the subject of which the predicate is asserted. For example, in the proposition *The earth is a planet* the subject is a *single* object; in the proposition *All planets move in elliptic orbits*, or *No planets are fixed*, the subject is a *class* or *kind*, and the assertion is made of *any* and every member of it; in the proposition *Some stars are* (or *are not*) *self-luminous* the subject is indeterminate, the assertion is made of one object at least, it may be of several, or of many objects, it may be of the whole class or kind. These differences are called differences of *quantity*. Propositions like the first of the above propositions are said to be *singular*; those like the second and third are called *general*; those like the last are called *particular*.

There are several minor points worth noting. (1) A proposition remains *singular* even when the subject of which the assertion is made consists not of a solitary object but of a group of objects, so long as the group is treated as a group, that is, as one complex object. For example, *The British Museum Library consists of several million books and pamphlets* is a singular proposition, because, although the subject is so numerous, the items are all regarded as forming one group, or collection. Similarly with the proposition *All the major planets are eight in number*. The subject is treated as one group, and the predicate

is asserted of the group as such, not of each of the planets separately. (2) As the last illustration may have suggested, it is important to distinguish carefully between a *group* and a *class* (or *kind*, or type). A *group* is a collection of several, or many, similar objects—multiplicity or *quantity* is an essential feature of a group; a class (in its scientific meaning, not as applied to school forms or standards), or kind, is so called in virtue of certain qualities or characteristics which distinguish it from other kinds of objects, etc. There are biological classes (or kinds) which are represented by the remains of a solitary specimen each; but mere number, or quantity, is of no consideration in the case of *kinds*. An assertion made of a class, or kind, as such, is made of that combination of qualities, and therefore of each thing that has those qualities—whether there are many such things, or only a few, or only one such thing, is immaterial. But an assertion made of a group may only be meant of the group as such, not of its individual components. Compare, for instance, the propositions *All planets move in elliptic orbits* and *All the major planets are eight in number*. The former is a general proposition concerning a class; the latter is a singular proposition concerning a group. (3) Singular and general propositions, though different from each other in the way indicated, have something important in common. In contrast with particular propositions they are both of them definite, or determinate. The particular proposition is inevitably indefinite, or indeterminate. For example, suppose I know that *Some stars are self-luminous*, from this alone I cannot tell whether the next star I observe, or hear of, is or is not self-luminous. That is the element of indefiniteness and

uncertainty. But both general and singular propositions are free from this. Hence they are usually grouped together as *universal* propositions—what they assert they assert of the whole extent of their subject (be that subject singular or innumerable), not of some indefinite part of it.

§ 4. *The Four Kinds of Categorical Propositions.*

If now we combine the main differences of quality and quantity considered above we obtain four main kinds of categorical propositions, namely, (1) *universal affirmative,* (2) *particular affirmative,* (3) *universal negative,* and (4) *particular negative.* These four kinds of categorical proposition may be represented respectively by the following formulæ: (1) *Every S is P,* if general (*This S is P,* if singular); (2) *Some S's are P*; (3) *No S is P* (*This S is not P,* if singular); and (4) *Some S's are not P.* It is convenient in some ways to have a brief designation for each of them, and so they are usually referred to, respectively, as (1) *A,* (2) *I,* (3) *E,* and (4) *O* propositions. These letters are derived from the Latin words *AffIrmo* (I affirm) and *nEgO* (I deny). And the symbolic formulæ for (1) *Every S is P,* (2) *Some S's are P,* (3) *No S is P,* (4) *Some S's are not P* are respectively (1) *SaP,* (2) *SiP,* (3) *SeP,* (4) *SoP.*

The meaning of these four propositional forms can be formulated in a perfectly unambiguous manner. There are other ways of expressing their respective meanings, and it is not necessary, or even desirable, to confine oneself pedantically to these four propositional forms. The important point to remember is that the other forms of statement are not so free from ambiguity, and, if and when any doubt may arise

about their precise meaning, it is best to express one's meaning in these unambiguous propositional forms. It is possible to express unambiguously any statement in these propositional forms, without doing violence to the King's English.

§ 5. *Relations between Terms in Categorical Propositions.*

Before proceeding to determine the precise meanings of the four propositional forms, there are some minor points to be cleared up. In different propositions (whatever their form may be—whether *A, I, E,* or *O*) the terms may be related in different ways. Sometimes the relation between the subject and the predicate is that between a thing (or things) and its (or their) attributes. For example, *Civilized people are tolerant, Oppressed people are discontented.* In other cases the relation is that between attributes. For instance, *Hopefulness induces cheerfulness, Despondency spells failure.* In still other cases the relation is that between classes (or kinds) of objects. For example, *The Norwegians are Scandinavians, Whales are mammals, Triangles are rectilinear figures.* And there are other relations possible, which need not detain us. Now for some purposes the differences between these relations may be of importance—say, from the point of view of psychology, or from the point of view of epistemology. But they are of no real importance for Logic, that is, for the study of valid inference. For our purpose no violence is done if, on occasion, a proposition expressing a relation between things and attributes, or a proposition expressing a relation between attributes, is so modified as to express a relation between classes or kinds. The main difference introduced is that of less abstractness (or more concreteness) of expression.

Logically there is no fundamental change involved, because classes or kinds (as already explained) are distinguished or characterized by their attributes. So, for example, there is no violence done if some of the above-mentioned propositions are restated as follows: *Civilized people are tolerant people, Hopeful people are cheerful people, Despondent people are people who fail.* This device of restating propositions expressed partly or wholly in terms of attributes in propositions expressed in terms of things, or classes, or kinds of things, has the advantage of simplifying the formulation of the meaning and implication of categorical propositions.

§ 6. *The Distribution of Terms in Categorical Propositions.*

Assuming, then, that the terms of categorical propositions may be legitimately interpreted in *extension*, that is, in terms of things, or classes of things (in contrast with *intension*, that is, in terms of attributes), we must next consider the question of the *distribution of terms* in categorical propositions. A term is said to be *distributed* when reference is made explicitly to its whole extent, that is, to the whole class of objects which it denotes; otherwise the term is said to be *undistributed*. In *E* propositions both terms are distributed—*SeP* means that the whole class *S* is outside of the whole class *P*. In *A* propositions the subject is distributed, but the predicate is undistributed—*SaP* means that the whole class *S* is included in the class *P*, but while the class *S* may sometimes coincide with the whole class *P* that is not always so, and must not be assumed to be so except on additional evidence. Compare, for instance, the proposition *All equilateral triangles are equiangular triangles,*

where the two classes coincide, with the proposition *All Danes are Scandinavians,* where the two terms do not coincide. In *O* propositions, the predicate is distributed, but the subject is undistributed—*SoP* means that one or more *S's* at least are outside the whole class *P*. In *I* propositions neither *S* nor *P* is distributed. The distribution of terms in categorical propositions may, therefore, be summed up as follows: *Only Universal Propositions distribute their subject, and only Negative Propositions distribute their predicate.*

§ 7. *General Rule of Formal Inference Concerning the Distribution of Terms.*

It is important to remember the distribution of terms in the four categorical propositional forms, if only because of a certain general rule which applies to all formal inference from given propositions (as distinguished from inductive inference from observation, etc.). The rule is this: *No term may be distributed in the conclusion unless it is distributed in the premise or premises.* The common sense of this rule is obvious. If a given term is not distributed in the premises, we have no evidence relating to the entire class which it denotes. But if the conclusion distributes that term, it asserts something about that whole class, and so goes beyond the evidence.

We may now proceed to consider the implications of the several forms of categorical proposition.

CHAPTER IV

IMMEDIATE INFERENCE—OPPOSITION

§ 1. *Kinds of Immediate Inference.*

By an *immediate inference* is meant whatever conclusion may be drawn from a single proposition, as distinguished from what may only be inferred from two or more propositions jointly. The problem of the determination of the various types of immediate inference falls into two parts. In the first part are considered the various inferences which may be drawn from a given proposition in terms, or in respect, of another proposition having the same subject and the same predicate as the given proposition, but differing from it in respect of quality, or of quantity. This part is known as *the doctrine of the opposition of propositions.* The second part deals with inferences which may be drawn from a given proposition involving certain other subjects and predicates than those of the given proposition. This part is known as *the doctrine of eductions.*

Both types of immediate inference, however, rest on certain fundamental assumptions which must first be considered briefly.

§ 2. *The Laws of Contradiction and Excluded Middle.*

The assumptions in questions are included among the so-called Laws of Thought, and are known as the

Law of Contradiction and the *Law of Excluded Middle.* According to the Law of Contradiction the same predicate cannot be both affirmed and denied of precisely the same subject—*S* (the same *S*) *cannot both be P and not be P*. According to the Law of Excluded Middle, a given predicate must either be affirmed or denied of a given subject—*S must either be P or not be P,* it cannot be neither, just as it cannot be both. These are fundamental assumptions on which all consistent thinking rests; all apparent exceptions rest on misunderstandings, or on quibbles. With the aid of these Laws of Thought we may now consider the opposition of propositions.

§ 3. *The Formal Opposition of Categorical Propositions.*

As already remarked, we are concerned here with the relations between propositions having the same subject and predicate, and differing only in form, that is, in quality or in quantity. Now there are only four such propositional forms—*SaP, SiP, SeP, SoP*—and we have to determine the relation of each to the others. Let us consider them in turn.

First *SaP.* According to it *P* is affirmed of every *S* without exception. Therefore if *SaP* is true *SiP* must be true; if it were not, that is, if it were possible not to affirm *P* of one or more *S's*, it would be impossible to affirm *P* of every *S*, that is, *SaP* could not be true. Again, *SaP* implies the falsity, or rejection, of *SeP*, for if *SeP* could be true at the same time as *SaP* then the same subject, each *S*, would at once *be* and *not be P*; and this would be a violation of the Law of Contradiction. Therefore, *SaP* implies the falsity of *SeP.* Similarly, *SaP* implies the falsity of *SoP.* For if both could be true together, then some *S's*

would both *be P* (because *SaP*) and *not be P* (because *SoP*); and this is excluded by the Law of Contradiction. Thus *SaP*, implies *SiP*, but excludes *SeP* and *SoP*.

Next *SiP*. *SaP* cannot be inferred from *SiP*, for the inference must not distribute a term (*S* in this case) not distributed in the premise; but, of course, *SaP*, though not inferrible from *SiP* may be true at the same time. Again, *SiP* excludes *SeP*, for if both could be true, then some *S's* would both *be P* (because *SiP*) and *not be P* (because *SeP*); and this would violate the Law of Contradiction. But *SiP* neither implies nor excludes *SoP*. It does not imply *SoP* because when *SiP* is true *SaP* may also be true, in which case *SoP* could not be true. It does not exclude *SoP*, because the "some *S's*" which are asserted to *be P* may be different *S's* from those which are asserted *not to be P*; and this would not involve a violation of the Law of Contradiction. They are, therefore, simply compatible; neither inferrible one from the other, nor exclusive one of the other.

Next *SeP*. This, as we have already seen, is inconsistent with both *SaP* and *SiP*. On the other hand, it implies *SoP* for the same reason that *SaP* implies *SiP*.

Lastly *SoP*. This, as we have already seen, is inconsistent with *SaP*, is compatible with *SiP*, and is implied by *SeP*.

So far we have considered the question whether these various pairs of propositions can or cannot be true at the same time. To complete our survey of their mutual relationships we must still consider whether they can or cannot be false at the same time. Or how does the falsity, or the rejection, of any one of

them affect the truth or falsity (the acceptance or the rejection) of the others?

Let us begin with *SaP*. Suppose it is not true. This means that it is incorrect to affirm *P* of every *S*—in other words, there are at least some *S's* (one or more) of which one should not say that it *is P*. But, according to the Law of Excluded Middle, anything must either *be P* or *not be P*. Consequently, of those *S's* (one or more) of which it is incorrect to assert that they *are P*, one must assert that they *are not P*, that is, *SoP*. Thus the falsity, or rejection, of *SaP* implies the truth, or the acceptance, of *SoP*. On the other hand, *SiP* and *SeP* may either of them be true (not both, of course) when *SaP* is false, one simply cannot tell.

Next take *SiP*. Suppose it is false. This means that it is incorrect to say of even one *S* that it *is P*. Consequently, according to the Law of Excluded Middle, it is right to say of every *S* that it *is not P*, or *SeP*. Thus the falsity, or rejection, of *SiP* involves the truth, or acceptance, of *SeP*, and therefore also of *SoP*, which, as already explained above, is implied by *SeP*. Obviously the falsity, or rejection, of *SiP* implies *a fortiori* the falsity, or rejection, of *SaP*—if it is incorrect to say of even one *S* that it *is P*, it must be even more incorrect to say of every *S* that it *is P*.

Consider next *SeP*. Suppose it is untrue. This means that it is incorrect to assert of every *S* that it *is not P*, or, in other words, that there is at least one *S* of which it is incorrect to assert that it *is not P*. If so, then, by the Law of Excluded Middle, it is correct to assert of at least that *S* that it is *P*, or *SiP*. Thus the falsity, or rejection, of *SeP* implies the truth,

or the acceptance, of *SiP*. On the other hand, it carries no implication with regard to *SoP* or *SaP*, either of which (though not both) may be true, or not, if *SeP* is not true.

Finally, *SoP*. Suppose it is false. This means that it is incorrect to assert of even one *S* that it *is not P*. Therefore, by the Law of Excluded Middle, it is correct to assert of every *S* that it *is P*, *SaP*. Thus the rejection, or falsity, of *SoP* implies the acceptance, or truth, of *SaP*, and therefore of *SiP*. It also implies the rejection, or falsity, of *SeP*—what cannot be asserted of *any S* can certainly not be asserted of *every S*.

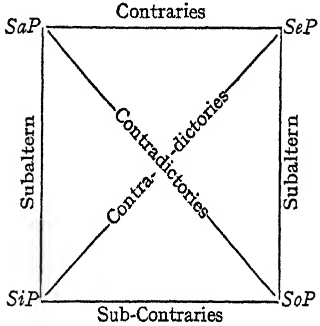
It may be helpful to sum up the foregoing results in the following table:

TABLE OF RELATIONS OR OPPOSITIONS.

Given	*SaP*	*SiP*	*SeP*	*SoP*
SaP true	—	true	false	false
SiP true	not known	—	false	not known
SeP true	false	false	—	true
SoP true	false	not known	not known	—
SaP false	—	not known	not known	true
SiP false	false	—	true	true
SeP false	not known	true	—	not known
SoP false	true	true	false	—

Some of the above relations between propositions having the same subject and predicate have received special names. They are summarized in the following diagram, which is known as

The Square of Opposition.



The following points should be specially noted:

Contraries. *SaP* and *SeP* are extreme opposites, and do not between them exhaust all possibilities. They cannot both be true; but they may both be false, namely, when both *SiP* and *SoP* are true. Singular propositions and particular propositions have no formal contraries, only contradictories.

Sub-contraries. The relationship between sub-contraries must be carefully distinguished from that between contraries, as they are the precise reverse of each other. Of *SiP* and *SoP*, one must be true, and both may be true—or both cannot be false, and neither need be.

Contradictories. Of the two propositions *SaP* and *SoP* both cannot be true, but one of them must be true. In other words, they are mutually exclusive (without being extreme opposites) and collectively exhaustive. The same holds good of *SeP* and *SiP*.

CHAPTER V

IMMEDIATE INFERENCE—EDUCTIONS

§ 1. *Eductions.*

In the doctrine of opposition we were concerned with the relations between propositions having precisely the same subject and predicate, but differing in quality or in quantity. We now proceed to consider the implications of the four types of categorical proposition beyond these limits. A proposition having *S* for its subject and *P* for its predicate may imply a proposition having *P* for its subject and *S* for its predicate, or propositions containing the contradictories of *S* and *P*. Such propositions are called *eductions*. There are two principal types of eduction, and five derivative forms. The principal types are known as the *converse* and the *obverse*; the other forms are obtained (when they can be obtained) by combining, or repeating alternately, the steps by which the converse and the obverse are obtained. The student who has no special distaste for dealing with a few easy symbols will find no difficulty with the Eductions. Above all he should endeavour to understand the steps by which the results are obtained, and not put his trust in merely remembering the results. Memory is a poor substitute for intelligence.

§ 2. *Contradictory Terms and their Symbols.*

It will be convenient at this point to say something about pairs of contradictory terms and their symbols, as the proper understanding of these simple matters is necessary to the real grasp of what follows.

Our thoughts and discussions usually have reference to some more or less well-defined class of objects, or qualities, etc., say, the inhabitants of the British Empire, economic goods, portraits, or colours, etc. Now all objects are what they are primarily because of the characteristics which they have, and the relations in which they stand; and for that reason all objects can be designated by means of positive terms, that is terms which imply or suggest what the objects are or have. But all finite objects, just because they are limited, are also without innumerable characteristics, for (as Spinoza says) *omnis determinatio est negatio* ("every limitation is a negation"); and for that reason they can also be referred to by means of negative terms, that is, terms which imply or suggest the absence of certain characteristics. No doubt it would be possible to refer to anything by means of positive terms. But there are such things as human needs and purposes, in relation to which some things, etc., are suitable while others are not. In this way many objects come to be designated by means of *negative terms*, that is, terms whose primary function it is to draw attention to what an object *is not*, or *has not*. For example, it may be necessary to estimate the war-strength of the British Empire, and for that purpose its inhabitants may be distinguished as *British subjects* (positive term) and *aliens* (negative term). Similarly, goods may be distinguished as *British* (posi-

tive term) and *foreign* (negative). It is obvious that the negative terms designate objects which have positive characteristics, just as the positive terms refer to objects which also lack all sorts of characteristics; and that positive and negative designations change places for different purposes. In France, for instance, British subjects are aliens, and British goods are foreign. Similarly, if one particularly wants a red tie, all other ties are simply "not red," whereas if one particularly wants a blue tie, blue ties cease to be regarded merely as "not red," and the red tie becomes merely "not blue." Now any pair of terms, by means of which a class of objects is divided into two mutually exclusive and collectively exhaustive classes, are called *contradictory terms.* Of a pair of contradictory terms one is *positive,* namely, the one which indicates the presence of the characteristic (or group of characteristics) in which one is interested at the moment, while the other is *negative,* namely, the one which indicates the absence of that characteristic. Sometimes the negative term is verbally expressed by prefixing "non" to the positive term—*non-productive, non-transferable, non-juror, non-payment, non-conformist non-commissioned officer, non-conductor,* etc. At other times the negative terms in use are not so constructed —for instance, *alien, foreign, brittle, opaque,* etc. But it is always permissible and possible to use a negative term of the former type in place of one of the latter type—for example, *non-British* (or *non-French,* etc.) for *alien,* or *foreign*; *non-flexible* for *brittle*; *non-transparent* for *opaque,* and so on.

Now a pair of contradictory terms when represented by symbols will naturally be represented in such a way that the symbols can at once be recognized as

contradictory. The simplest way to this end would appear to be to represent the positive term by a positive symbol, say *S*, *P*, etc., and the negative term by a negative symbol, *non-S*, *non-P*, etc., or, more conveniently, $\bar{S}$ (= *non-S*), $\bar{P}$ (= *non-P*), etc. This is always possible. Sometimes it is also convenient; but not always. It is usually more convenient to represent the terms of a given proposition by means of positive symbols, even when those terms are negative. It is more convenient because it is easier in that case to see at a glance the relation of the given proposition to other propositions (immediate inferences, etc.). And there is no objection to doing so. All one has to remember is that just as in Algebra x, y, z, etc., may represent negative quantities, so in Logic *S*, *P*, etc., may represent negative terms. The main thing is that *S* and $\bar{S}$, *P* and $\bar{P}$, etc., are pairs of contradictory terms. But *S* and $\bar{S}$ are just as contradictory when *S* represents the negative term and $\bar{S}$ the positive term as when *vice versa*.

§ 3. *Alternative Formulation of the Laws of Contradiction and of Excluded Middle.*

In the light of what has just been said about contradictory terms, it is also possible now to give alternative formulations to the Laws of Contradiction and Excluded Middle. It will be convenient to give these alternative formulations at this stage because they will be helpful in connection with the doctrine of Eductions which will be discussed next.

The formula for the Law of Contradiction, as given in the preceding chapter, is *S cannot both be P and not be P*, and the formula for the Law of Excluded Middle is *S must either be P or not be P*. As already explained

in an earlier part of the book, what happens in a categorical proposition may be described thus: some subject is related in some way to a class. Let S be the subject and Q the class. Then the class Q can usually be conceived as consisting of two mutually exclusive, and collectively exhaustive, sub-classes, say P and $\bar{P}$, according to the purpose of the thinker. The sub-classes P and $\bar{P}$ being related as they are, if Q is relevant at all in the consideration of S, then S is either in sub-class P or in sub-class $\bar{P}$, and cannot be in both. We may accordingly also express the *Law of Contradiction* in the formula, *S cannot be both P and $\bar{P}$*; and the *Law of Excluded Middle* may be expressed in the formula *S must be either P or $\bar{P}$*.

Armed with these formulæ we may now proceed to consider the different kinds of eductions. And first of all the fundamental types of eduction which are known as Obversion and Conversion.

§ 4. *Obversion.*

The obverse of a given proposition is a proposition implied by it, having the same subject but a contradictory predicate. Symbolically, if the terms of the given proposition are $S—P$, those of its obverse will be $S—\bar{P}$. Every type of categorical proposition has an obverse of opposite quality. Thus SaP implies $Se\bar{P}$, for, by the Law of Contradiction, if every S is P no S can be $\bar{P}$. Similarly, SiP implies $So\bar{P}$, for, by the same law, the one or more S's which are P cannot be $\bar{P}$. Again, SeP implies $Sa\bar{P}$, for, by the Law of Excluded Middle, S must be either P or $\bar{P}$, therefore if no S is P every S must be $\bar{P}$. Similarly, SoP implies $Si\bar{P}$, for, by the same law, the one or more S's which are not P must be $\bar{P}$.

It should be noted that a proposition and its obverse are equivalent, or reciprocal, that is, each implies the other. We have already seen, and we shall see again in due course, that a proposition may imply another proposition which does not imply it in return. For example, SaP implies SiP, but SiP does not imply SaP. Similarly, SeP implies SoP, but SoP does not imply SeP. The sign " $=$ " should be reserved for equivalent propositions, and a single arrow, pointing in the right direction, for one-sided implication. Thus $SaP = Se\bar{P}$, $SiP = So\bar{P}$, $SeP = Sa\bar{P}$, $SoP = Si\bar{P}$; but $SaP \rightarrow SiP$, $SeP \rightarrow SoP$.

Another point to be noted is that if the obverse of a given proposition is itself obverted we simply get back to the original proposition. So that no new result can be obtained by merely repeating the process of obversion. This should be remembered when dealing with the more complex eductions.

§ 5. *Conversion.*

The converse of a given proposition is a proposition implied by it, but having its subject for predicate, and its predicate for subject. In other words, the terms of the given proposition and those of its converse (if it has a converse) are related as $S-P$ to $P-S$. Not every proposition, as we shall see soon, has a converse.

It has already been explained in an earlier part of the book, that the subject of a proposition is the comparative starting-point in thought, it is what is felt to need explanation, or elucidation. Now what is the starting-point in thought on one occasion, or to one person, may not always be so. S may be the starting-point on one occasion, and P its elucidation;

but P may be the starting-point on another occasion, and S its elucidation. In geometry, for example, we at one stage try to find out whether "equilateral triangles are equiangular" ($S—P$), at another stage we want to ascertain whether "equiangular triangles are equilateral" ($P—S$), and so on. The question we have to consider here is simply this. Given a proposition in which something is asserted about S in terms of P, what does it imply about P in terms of S, by way of immediate inference, that is, in the absence of any other evidence?

Let us begin with SeP. This may be said to mean that the whole class S is outside of, or different from, the whole class P. If we express precisely the same relation from the point of view of P, instead of from the standpoint of S, we get PeS. Thus $SeP = PeS$. Again, SiP means that some S's are included in the class P, and this means that they are identical with some P's. What we really have, then, are objects which are both S and P, and which can therefore be described indifferently as some S's which are P, or as some P's which are S. Thus $SiP = PiS$. It will thus be seen that E and I propositions can be converted simply—one may just transpose their terms without changing in any other way the character of these propositions. It is different with A and O propositions. SaP means that the whole class S is included in the class P, that is, is identical with some indefinite part of the class P. The two classes may occasionally coincide, as for instance in the statement *All equilateral triangles are equiangular triangles*, but they may not, as, for example, in the statement *All rectangles are parallelograms*. What we always have when SaP is true is a class of objects which are both S and P,

which consists of the whole class S, but may not consist of all P's, though it must include at least some P's. P, as we have seen in a preceding chapter, is undistributed here. Therefore, in the absence of additional evidence, P may not be distributed in the conclusion, that is, we must not make *SaP* imply *PaS*, but only *PiS*. Thus *SaP* $\Rightarrow$ *PiS*. This is called conversion by limitation, that is, by limiting, or restricting, the (universal) quantity of the original proposition—in contrast with *simple* conversion, where the given proposition and the converse have the same quantity. Lastly, *SoP* has no converse at all. To convert *SoP* into *PoS* would be to distribute, in the inference, a term (S) undistributed in the premise, and that is not permissible. The fact is that *PaS* may be true at the same time as *SoP*. For example, *Some rectangles are not squares*, yet *All squares are rectangles*, or *Some Europeans are not Swedes*, but *All Swedes are Europeans*. Hence *SoP* cannot imply *PoS*, which is the contradictory of *PaS*. Thus *SoP* has no converse.

It should be noted that when the converse of a given proposition is itself converted then in the case of E and I propositions we simply come back to the original proposition, while in the case of A propositions even that much is not achieved, for we get *SiP* instead of *SaP*. Therefore, in the case of conversion, as in the case of obversion, no new result can be obtained by merely repeating the process. The only way of obtaining new eductions is by applying the processes of obversion and conversion alternately, each to the result of the other. That, indeed, is the way in which the other eductions are obtained.

Before proceeding farther it may be helpful to tabulate the eductions dealt with so far.

§ 6. *Table of Principal Eductions.*

TABLE OF PRINCIPAL EDUCTIONS.

Original $S—P$	Obverse $S—\overline{P}$	Converse $P—S$
SaP	$Se\overline{P}$	PiS
SiP	$So\overline{P}$	PiS
SeP	$Sa\overline{P}$	PeS
SoP	$Si\overline{P}$	None

CHAPTER VI

IMMEDIATE INFERENCE—DERIVATIVE EDUCTIONS

§ 1. *Conceivable Eductions.*

In the preceding chapter it was shown that all propositions imply an obverse the terms of which are related to the terms of the original as $S-\bar{P}$ to $S-P$, and that some propositions have a converse, the terms of which are related to those of the original as $P-S$ to $S-P$. Let us now consider in a purely abstract manner what other implications are conceivable involving the terms of a given proposition and their contradictories. The terms in question will be four in number, namely S, P, $\bar{S}$, $\bar{P}$, of which S and P represent the subject and predicate of the given proposition. Let us omit merely tautological statements, such as *S is S*, or *S is not* $\bar{S}$, and self-contradictory statements like *S is not S*, or *S is* $\bar{S}$. We are then left with the following conceivable combinations of terms for conceivable additional eductions, namely, $P-\bar{S}$, $\bar{P}-S$, $\bar{P}-\bar{S}$, $\bar{S}-P$, $\bar{S}-\bar{P}$. If we add to these the original $S-P$, the obverse $S-\bar{P}$, and the converse $P-S$, we obtain the following table of conceivable combinations of terms of propositions in relation to any given proposition. As there can be no harm in christening these various conceivable

combinations before studying them more closely, names are added in all cases.

1. $S-P$ original proposition.
2. $S-\bar{P}$ obverse.
3. $P-S$ converse.
4. $P-\bar{S}$ obverted converse.
5. $\bar{P}-\bar{S}$ contrapositive.
6. $\bar{P}-S$ obverted contrapositive.
7. $\bar{S}-P$ inverse.
8. $\bar{S}-\bar{P}$ obverted inverse.

Of these combinations it will be seen that Nos. 4, 6, and 8 are each related to its immediately preceding combination (Nos. 3, 5, and 7 respectively) in exactly the same way as the obverse (No. 2) is related to the original proposition (No. 1). They are accordingly called each the obverse of the preceding form (obverted converse, etc.). If the converse can be obtained, then there is no difficulty in getting its obverted form, since every proposition, as we have seen, has an obverse. Similarly, if and when the contrapositive and inverse forms (Nos. 5 and 7) can be obtained, there will be no difficulty in determining their obverted forms (Nos. 6 and 8).

§ 2. *Actual Derivative Eductions.*

We have now to ascertain which of these conceivable eductions are implied by, or can be derived from, the four types of categorical propositions. The obverse and the converse have already been dealt with, and the results are tabulated at the end of the foregoing chapter. Keeping an eye on that table, it will be seen that by obverting the converse we obtain the following results: $SaP \rightarrow PiS = Po\bar{S}$; $SiP = PiS = Po\bar{S}$;

and $SeP = PeS = Pa\bar{S}$. Similarly by converting the obverse we obtain the following contrapositives (No. 5): $SaP = Se\bar{P} = \bar{P}eS$; SiP has none, because its obverse ($So\bar{P}$) being an O proposition cannot be converted; $SeP = Sa\bar{P} \longrightarrow \bar{P}iS$; $SoP = Si\bar{P} = \bar{P}iS$. By obverting the contrapositives we get the obverted contrapositives (No. 6) as follows: $SaP = \bar{P}a\bar{S}$; $SeP \longrightarrow \bar{P}o\bar{S}$; $SoP = \bar{P}o\bar{S}$. It only remains now to determine the inverse forms (Nos. 7 and 8). These forms can only be obtained by converting either the obverted converse (No. 4) or the obverted contrapositive (No. 6). Now the obverted converse of SaP and SiP being O propositions cannot be converted; that of SeP is $Pa\bar{S}$ which converts into $\bar{S}iP$; SoP having no converse has no obverted converse. Again, the obverted contrapositive of SaP is $\bar{P}a\bar{S}$ which converts into $\bar{S}i\bar{P}$, and this obverts into $\bar{S}oP$; the obverted contrapositive of SoP is $\bar{P}o\bar{S}$, and cannot therefore be converted; SiP has no obverted contrapositive; and SeP has already been dealt with. Thus only the two universals have an inverse, namely, $SaP \longrightarrow \bar{S}oP$, and $SeP \longrightarrow \bar{S}iP$. And they, of course, also have an obverted inverse, namely, $SaP \longrightarrow \bar{S}i\bar{P}$, and $SeP \longrightarrow \bar{S}o\bar{P}$. We may now tabulate the complete list of eductions. The table looks rather formidable at first sight, but is really quite easy to remember, as will be explained soon. The complete Table of Eductions is given on the next page.

COMPLETE TABLE OF EDUCTIONS.

1 Original Proposition $S—P$	2 Obverse $S—\bar{P}$	3 Converse $P—S$	4 Obverted Converse $P—\bar{S}$	5 Contrapositive $\bar{P}—S$	6 Obverted Contrapositive $\bar{P}—\bar{S}$	7 Inverse $\bar{S}—P$	8 Obverted Inverse $\bar{S}—\bar{P}$
SaP	$Se\bar{P}$	PiS	$Po\bar{S}$	$\bar{P}eS$	$\bar{P}a\bar{S}$	$\bar{S}oP$[1]	$\bar{S}i\bar{P}$
SiP	$So\bar{P}$	PiS	$Po\bar{S}$	None	None	None	None
SeP	$Sa\bar{P}$	PeS	$Pa\bar{S}$	$\bar{P}iS$	$\bar{P}o\bar{S}$	$\bar{S}iP$	$\bar{S}o\bar{P}$
SoP	$Si\bar{P}$	None	None	$\bar{P}iS$	$\bar{P}o\bar{S}$	None	None

[1] As this inverse of *SaP* distributes *P*, which was not distributed in the original, we seem to have here a violation of the general rule of formal inference stated on p. 37. The explanation is as follows. If everything in the universe were *P*, there would be no point in affirming *P* of just *S*. The assumption underlying the assertion *SaP* is that *Some things are not P*. Now this underlying assumption, or auxiliary proposition, distributes *P*. And this is the justification for the distribution of *P* in $\bar{S}oP$, the inverse of *SaP*.

§ 3. *Complete Table of Eductions.*

It is unnecessary and undesirable to commit this table of eductions to memory. The only things really necessary are these: to master thoroughly the processes of obversion and of conversion; to remember that any one of the other eductions, if it can be obtained at all, can be obtained by *alternate* obversion and conversion, so that if it cannot be obtained by beginning with obversion, one should begin with conversion, and if it cannot be got then, it is not obtainable; lastly one has to remember what combination of terms each name denotes. For this last purpose the following device should be sufficient. Commit to memory:

S	P	$\bar{P}$	
Obverse	*Converse*	*Contrapositive*	*Inverse*

The symbols represent the *subjects* of the forms required. If S is the subject of the obverse, the predicate must be $\bar{P}$—not P because $S-P$ is the original. If P is subject of the converse, S will be the predicate of the converse, $\bar{S}$ of the obverted converse. Similarly with $\bar{P}$ and $\bar{S}$ as the subject of the contrapositive and of the inverse respectively.

CHAPTER VII

OTHER IMMEDIATE INFERENCES

In addition to the formal *oppositions* and *eductions* already described, there are certain other kinds of immediate inference which may be considered now. They are (*a*) *material opposition*, (*b*) *converse relation*, or *correlation*,[1] and (*c*) *complication of terms*.

§ 1. *Material Opposition*.

In the doctrine of opposition, as treated in Chapter IV, we were concerned with various relations between propositions having precisely the same subject and the same predicate, and differing only as regards *form*, that is, in respect of quality or quantity. But relations essentially similar to those of contrariety, contradiction, sub-contrariety, and subalternation may arise from the nature of the relations between the terms employed (the form of the proposition remaining the same), or from a combination of the relations partly between the terms and partly between the forms of the propositions. In so far as these relationships (or "oppositions") do not depend entirely on the *forms* of the propositions, but

[1] Not to be confused with statistical correlation, which is something entirely different. (See *Essentials of Scientific Method*, Ch. VI.)

wholly or partly on their terms, they may be described as *material oppositions,* while those dealt with in Chapter IV may be called *formal oppositions,* since they depend entirely on the forms of the propositions in question, the terms being the same.

Material Contraries. (i) Two propositions may be contraries because they affirm contrary predicates of the same subject. Two terms are said to be contrary when they mean extreme opposites within the same universe of discourse, and so do not exhaust it. Thus, for example, in the universe of colour, *white* and *black* are contraries; in the realm of morality, *saintly* and *wicked*; in the sphere of property, *wealthy* and *destitute,* and so forth. Now two *universal* propositions in which contrary predicates are *affirmed* of the same subject are material contraries. Like formal contraries they cannot both be true, but may both be false. What is essentially the same result occurs also when the two predicates, instead of being contrary terms, are mutually exclusive but not collectively exhaustive species of the same genus, like *white* and *brown, red* and *green, isosceles* and *scalene,* and so on.

(ii) Again, after what has been said about obversion, in Chapter V, it should be obvious that *general* propositions in which contradictory predicates are asserted of the same subject are contraries. For SaP is related to $Sa\bar{P}$ exactly as to its equivalent SeP; and SeP is related to $Se\bar{P}$ exactly as to its equivalent SaP.

Material Contradictories. (i) Two singular propositions in which contradictory terms are asserted of the same subject are material contradictories. Both propositions cannot be true, but one must be.

(ii) The same applies to two propositions of opposite quantity, but of the same quality in which contradictory predicates are asserted of the same subject-term. For *SaP* is related to $Si\bar{P}$ exactly as to its equivalent *SoP*; and *SeP* is related to $So\bar{P}$ in the same way as to its equivalent *SiP*.

Material Sub-Contraries. Obviously *SiP* is related to $Si\bar{P}$ in the same way as to *SoP*; and *SoP* is related to $So\bar{P}$ in the same way as to *SiP*.

Material Subalternation. Suppose we have a proposition of the type *SaP* in which *S* is a general term including two or more sub-classes, say, S_1, S_2, etc. then *SaP* obviously implies S_1aP, S_2aP, etc., while S_1aP, or S_2aP, etc., does not imply *SaP*. For instance, *Scandinavians are Europeans* implies *Danes are Europeans*; *Parallelograms have their opposite sides equal* implies *Rectangles have their opposite sides equal.* Similarly with *SeP* in relation to S_1eP, S_2eP, etc. For example, *Scandinavians are not of Mediterranean race* implies *Swedes are not of Mediterranean race.* Again, suppose that *P* is a sub-class (or species) of a more general term (or genus), say *Q*, then *SaP* implies *SaQ* because the attributes implied by *P* (the specific term) include the attributes implied by *Q* (the generic term). Thus *Cornishmen are Englishmen* implies *Cornishmen are British.* On the other hand *SaQ* does not imply *SaP*. The relation of *SaP* to S_1aP, or S_2aP, etc., and of *SeP* to S_1eP, or S_2eP, etc., also of *SaP* to *SaQ*, is that of subalternation, or that of *subalternant* (the proposition which implies another without being implied by it) to that of *subalternate* (the proposition which is implied by another without implying it). But the relation between *SeP* and *SeQ* is the reverse of that between *SaP* and *SaQ*—*SeQ* implies *SeP*,

not *vice versa.* So, for instance, *No Indians are Englishmen* does not imply *No Indians are British,* but *No Egyptians are British* would imply *No Egyptians are English.*

§ 2. *Immediate Inference by Converse Relation.*

In some propositions the predicate (P) asserts some definite relation (say, R) in which the subject (S) stands to a certain term (say, Q). In such cases we can deal with the predicate (P) as a whole, and so obtain the ordinary converse of the proposition ($P—S$), etc., or we may analyse the predicate into its components (R, Q) and infer that since S stands to Q in the relation R, Q must stand to S in the converse relation (or correlation) *Я*, where R and *Я* represent a pair of correlative terms, that is, terms which are specially given to draw attention to a definite relationship between certain objects, like *north* and *south, right* and *left, teacher* and *pupil,* etc. Take the proposition *S is due north* (R) *of* Q. The ordinary converse would be *Some place due north of* Q *is S* (*PiS*). But by converse relation it also implies Q *is due south of S* (Q *is ЯS*), which may be called its *correlative proposition.* Similarly the proposition *University professors are teachers of University students* implies, as its *converse, Some teachers of University students are University professors,* but its *correlative* is *University students are pupils of University professors.* Inference by converse relation (or correlation) is very common in ordinary life. It frequently happens that we want to know the relation of Q to S, but, if S is the more important of the two, the works of reference we consult will give us information about the relation of S to Q. This may serve our purpose just as well,

because we can formulate for ourselves the required correlative.[1]

§ 3. *Immediate Inference by Complication of Terms.*

Given the proposition *S is P* it is possible and sometimes convenient to add a third term to both *S* and *P*, say *D*, and so obtain the inference *DS is DP* the terms of which are more complex than those of the original proposition. For example, the proposition *Triangles are figures* implies the propositions *Plane triangles are plane figures*, *Equilateral triangles are equilateral figures*, *Spherical triangles are spherical figures*, *Combinations of triangles are combinations of figures*, *The study of triangles is the study of figures*, *The symbolic use of triangles is the symbolic use of figures*, and so on. Some distinguish two varieties of this kind of immediate inference, namely, that *by added determinants* and that *by complex conception*, according as *D* is a determin*ant* of *S* and *P* (that is, restricts or qualifies them), or a determin*atum* of *S* and *P* (that is, is restricted or qualified by them). The first three of the above examples illustrate immediate inference by added determinants; the last three illustrate immediate inference by complex conception. But it is simpler to treat them both under a common designation, for they are alike essentially.

The validity of immediate inference by complication of terms depends on two conditions: (i) The deter-

[1] That the mutual implication of correlative propositions is not always appreciated appears from the following story. A lady, it is related, told her shoemaker that one of her feet was bigger than the other. "Quite the contrary, madam," said the gallant shoemaker. "I find that one of your feet is smaller than the other."

minant or determinatum (*D*) must be relevant to the terms of the original proposition (*S* and *P*); (ii) it must have precisely the same meaning in conjunction with both terms. (i) If it is not relevant, the result will be sheer nonsense. (ii) If it has not the same meaning in the two cases it is not really the same determinant or determinatum.

This form of inference, where it is employed, is rarely set out explicitly. The "given proposition" (*S is P*) is tacitly assumed rather than clearly formulated, and the conclusion usually takes the form of substituting one term for another. One speaks of a strong *poison*, or of a dose of *poison*, instead of a strong *arsenic*, or a dose of *arsenic*, for instance, on the implicit ground that *arsenic is a poison, S is P*, therefore *DS is DP*. But mistakes are all the more apt to arise when the ground of the inference is not stated explicitly. The chief source of mistakes is to be found in what may be called the *relativity* of certain terms expressing quantity or degree. Some terms may be said to express such values absolutely—for example, *an English mile*, 100° *Centigrade*, £100 *sterling*, and so on. There are other terms which express such values not absolutely, but relatively—for instance, *far*, *near*, *hot*, *warm*, *rich*, *poor*, *big*, *small*, etc. Such relative terms of measure are apt to suggest, or even imply rather different values in different contexts, and so involve occasionally a breach of the second condition stated above. Thus, for instance, it would be misleading to describe even the smallest elephant as "a tiny animal" merely on the ground that *an elephant is an animal*; or to describe ice-pudding as "a warm dish" merely because *it is a dish* and it is not as cold as it should be; or to describe a dock-labourer in

receipt of a considerable unemployment dole as "a rich man," merely because *a dock-labourer is a man,* and he is comparatively well off for his station and under the circumstances. Similarly it would be false to describe the views of the majority of barristers, or of head masters, as the views of the majority of lawyers, or of teachers, respectively, merely because *barristers are lawyers,* and *head masters are teachers.*

CHAPTER VIII

MEDIATE INFERENCE FROM PARTICULARS

§ 1. *The General Character of Mediate Inference.*

So far we have considered the inferences that may be drawn from single categorical propositions. We must determine next what inferences may be drawn from two or more of them jointly, over and above the immediate inferences implied by each of them separately. It is obvious that two or more propositions involving entirely different terms cannot between them imply anything more than the sum of their separate implications. But where two propositions have a term in common, then something may be inferrible from the two together which could not be inferred from either separately, and which is not merely the sum of their separate implications. This common term may mediate between the other terms of the two propositions so as to establish a relationship between them that could not be inferred from either proposition alone. That is of the essence of mediate inference—given the relationship of each of two terms to the same third term, it is possible under certain conditions to infer their relationship to each other. Symbolically, if we know how S is related to M, and how P is related to M, it may be possible to infer how S is related to P.

A story related of an incident which is said to have

happened at a reception given by a French Countess may serve as an illustration of mediate inference. Among the early callers was a certain Cardinal, in conversation with whom his hostess remarked sympathetically on the wide and varied experience he must have had. The Cardinal assented, adding that he had, in fact, started rather badly, for the very first person to confess to him had confessed a murder! Some time afterwards, while the Cardinal was conversing with some one at the far end of the *salon*, a well-known French Count called, and the hostess, after chatting with him a while, suggested that she would like to introduce him to the Cardinal. The Count replied that no introduction was necessary as he had known His Holiness many years. "In fact," added the Count, "I was the very first person to confess to him, and (he added with a twinkle in his eye) let me assure you, Countess, that my confession did surprise him!" The feeling of the hostess may easily be imagined.

§ 2. *Mediate Inference with a Singular Middle Term.*

In the foregoing illustration the middle term is singular (that is, denotes an individual object), and the relation between the terms is that of identity. When that is the case mediate inference is easy and obvious. The identity of "the Count" with "the first person who confessed to the Cardinal," and of this person with "a person who confessed a murder," implies the identity of "the Count" with the "person who confessed a murder." It is evident that if two terms (S and P) are each identical with the same singular third (or middle) term (M), then they must be identical with each other. M *is* P, S *is* M, $\therefore$ S *is* P.

S being identical with *M*, they are really different names of the same thing, so that what is called *M* may also be called *S*. Now, if it were possible in this case for *S* not to be *P*, then the same thing (called indifferently *S* or *M*) would be *P*, according to the first premise, and would not be *P*, if the suggested conclusion were rejected. But that would violate the Law of Contradiction. The conclusion, *S is P*, is therefore valid. So far only affirmative propositions have been considered as the premises of mediate inferences. Is it possible to draw mediate inferences also from negative premises? A distinction must be drawn between cases in which both premises are negative, and those in which one premise is negative, and the other is affirmative.

When both premises are genuinely negative, and not merely the obverse equivalents of affirmative propositions, then they do not warrant any mediate conclusion. From *S is not M* and *M is not P*, no inference can be drawn with reference to *S* and *P*. There is no real mediation in such cases, for we are not really told how *S* and *P* *are* related to *M*, only how they are *not* related to it; and their common difference from *M* in this one respect is compatible with the relation either of identity or of difference between *S* and *P*. Cases like *S is not* $\overline{M}$ and *M is not* $\overline{P}$, ∴ *S is P* are no real exceptions because the premises are merely obverse forms of *S is M* and *M is P*, and since *M* is assumed to be singular, the conclusion is legitimate. But if both premises are genuinely negative, that is, genuine negations, then no mediate inference is justified.

If, on the other hand, one premise is affirmative, and the other is negative, then it is sometimes legitimate to infer a conclusion, but not always. Let the relation-

ship asserted in the affirmative proposition be that of identity, then the negative premise will in that case deny identity, or, what is the same thing, assert difference. *S* and *P* will in that case be related in contradictory ways to the same third term, *M* (which is still assumed to be a singular term), and must consequently be different from each other in some respect, that is, *S is not P*. Thus *S is M* and *M is not P* imply *S is not P*; and *S is not M* and *M is P* also imply *S is not P*. In the first example *S* and *M* are identified in the first premise, and so *S* may be substituted for *M* in the second premise, and then the Law of Contradiction would be violated if the conclusion were *S is P*. Similarly, in the second example, *M* is identified with *P* in the second premise, and so *P* may be substituted for *S* in the first premise, and then the Law of Contradiction would be violated if the conclusion were *S is P*.

§ 3. *Identity and other Transitive Relations.*

In the foregoing account of mediate inference with singular middle terms the premises were all such as expressed relations of identity and difference. But some of the results arrived at are applicable also to all other transitive relations. A transitive relation is such that if it hold good between one term and a second term, and likewise between the second term and a third term, then it will also hold good between the first term and the third term. Identity is one such relation—if *S* is *M* and *M* is *P*, then *S* is *P*, as just explained. Equality is another transitive relation—if $S = M$, and $M = P$, then $S = P$. The relations of "greater than" and "less than" are also transitive—if $S > M$ and $M > P$, then $S > P$.

But there is no general way of indicating how transitive relations may infallibly be distinguished from others, and care has to be exercised. For example, while "brother of," "sister of," "ancestor of" are transitive relations, "parent of," "uncle of," "brother-in-law of" are not transitive relations. So long, however, as the relation is transitive, and the middle term is singular, the two propositional forms *SrM*, *MrP* will imply *SrP*—when *r* stands for the assertion of the relation in question.

When both premises are affirmative there is, then, no essential difference between mediate inference from singular premises which express relations of identity and those which express other transitive relations.

Similarly, when both premises are negative, no valid conclusion can be drawn in either case. From *S is not r M* and *M is not r P* nothing can be inferred with regard to the relation between *S* and P. Their common difference from *M* may be compatible with all sorts of relations between *S* and *P*.

It is entirely different when one premise is affirmative and the other is negative. With some kinds of transitive relations, such as equality or parallelism, for instance, the case is just like that of identity, and a negative conclusion follows. Thus, for example, if *S* and *P* are one of them equal to *M* and the other not, then *S* is not equal to *P*. But it is different with other relations, even transitive relations. From *S is greater than M* and *M is not greater than P* no inference can be drawn, while *S is greater than M* and *P is not greater than M* imply that *S is greater than P*.

Unfortunately no general rules can be laid down for propositions which do not express relations of identity, or which cannot be so interpreted as to

express relations of identity (or difference, when the propositions are negative).

The relation of identity (and its negation, difference) is, however, a very comprehensive one, and other relations are sometimes reducible to it. Take, for instance, the relation of equality. It is only a more concrete way of expressing a more abstract identity of some quantity or other. For example, let S, M and P stand for lines. Then $S = M, M = P, \therefore S = P$ may be restated just as accurately, or even more so, in terms of identity. Thus: *The length of S is the length of M, the length of M is the length of P, ∴ the length of S is the length of P.* Similarly, the argument *S is as rich as M*, and *M is as rich as P, ∴ S is as rich as P*, may be restated in the following statements of identity. *The value of the possessions of S is the value of the possessions of M, The value of the possessions of M is the value of the possessions of P, ∴ The value of the possessions of S is the value of the possessions of P.* (Needless to say, it is not suggested that S, M and P are identical in these cases, in which S, M and P are not the whole predicates, but only parts of them.) With a little ingenuity other relations can likewise be expressed in terms of identity; but it would take us too far afield to consider them here. Enough that a great number of the premises with the implications of which one is mostly concerned either are, or can be, expressed in terms of identity and difference.

§ 4. *Dovetail Relations.*

Besides transitive relations there are also certain other relations which frequently occur in mediate inference. These may be called *dovetail* relationships, for the following reason. In the case of mediate

inference involving transitive relations what usually happens is that the premises attest that the major term and the minor term are each related in the same kind of way to the same middle term, and the conclusion states that the same relationship therefore holds good between the major and minor terms themselves. The cases in which one premise is negative are not essentially different—they also refer to the same relationship throughout, which is affirmed in one premise, denied in the other, and usually denied in the conclusion. It is different with the cases now under consideration. In these cases one premise may assert one kind of relationship, the other a different relationship, and in the conclusion yet a third relationship is obtained by dovetailing the other two relationships, in the light of our knowledge of the system of relationships involved. An example or two will make this clear.

(1) *M is the brother of P,*
S is the son of M,
∴ *S is the nephew of P.*

(2) *M is due north of P,*
S is due west of M,
∴ *S is north-west of P.*

It should be realized that inferences involving other relations than those of identity and difference (or relations reducible to these) cannot be made safely without a knowledge of the actual system of relationships involved in each case. They are not so formal as those involving only relations of identity and difference. Hence they are usually ignored in Formal Logic. But the mediate inference involved is essentially of the same character throughout, and the systems

of relationships mostly concerned are such that intelligent people are generally conversant with them.

§ 5. *Rules of Mediate Inference with a Singular Middle Term.*

The results reached so far, with regard to mediate inference from particulars, may be summarized as follows:

(1) If both premises are irreducibly negative (that is, are not merely the obverses of affirmative propositions), then no mediate inference is justified.

(2) If the middle term is singular, and both premises are affirmative, and express a relation of identity, or some other transitive relation, then it is permissible to draw an affirmative conclusion with the other two terms of the premises. Symbolically: *S is M, M is P,* ∴ *S is P,* or, more generally, *SrM, MrP,* ∴ *SrP,* where *r* represents the affirmation of any transitive relation.

(3) If the middle term is singular, and the premises are one affirmative and one negative, and express a relation of identity and difference repectively, then a negative conclusion is inferrible. Symbolically: *S is M, M is not P,* ∴ *S is not P*; or *S is not M, M is P,* ∴ *S is not P.*

Conversely, to draw a negative inference one premise must be affirmative, and one negative. This follows from (1) and (2).

In this case, unlike (1), it is not possible to extend the formulæ so as to include *all* transitive relations; but some such relations, for instance, those of equality and parallelism, can be treated in the same way as identity.

CHAPTER IX

MEDIATE INFERENCE WITH A GENERAL PREMISE

§ 1. *Complications Arising when the Middle Term is General.*

(*a*) We may now consider the conditions of mediate inference when the middle term is not a singular term but general (or a class name). In this case certain complications arise. So long as the middle term (*M*) is unambiguous and singular the main feature of mediate inference is obvious, and its principal condition is satisfied, namely, that it is really the same third term with which the other two terms (*S* and *P*) are compared. It is obvious that if the two premises involve entirely different terms, say *S* and *M*, in one case, and *Q* and *P*, in the other, then no inference can be drawn about *S* and *P*. Now the mere fact that the same *general name* occurs in two propositions does not necessarily mean that the same *things* are referred to. The members of the class referred to in one proposition may be quite distinct from those referred to in the other proposition, and so there may be no real mediating link. For example, suppose we have the premises, *All Danes are Scandinavians*, and *Some Scandinavians are Norwegian.* It is obvious that the Scandinavians mentioned in the first proposition are not the same as those mentioned in the second. Danes and Norwegians are consequently not identified with

the *same* third term, and it would therefore be erroneous to identify them with one another. Under what circumstances, then, when the middle term is general, can we be sure that the two premises, in both of which it occurs, really have the same middle term? The answer is, when the middle term is distributed in at least one of the two premises. For, if one premise refers to the whole of the class denoted by the middle term, then whatever part of that class the other premise may refer to, the two premises are bound to have something really in common. For example, the two propositions, *All Danes are Scandinavians*, and *All Scandinavians are Europeans*, do imply that *All Danes are Europeans*. Here the middle term (Scandinavians) is distributed in the first premise, though not in the second. Consequently the Scandinavians referred to in the second premise are identical with some of those referred to in the first, and the fact that the Danes and some Europeans are each identified with the same Scandinavians, implies the identity of the Danes with some Europeans. When the middle term is singular it is not necessary to say anything about its distribution, because it is inevitably distributed, as it denotes an individual object and cannot denote less. But the requirement that the middle term should be distributed in at least one of the premises really is a rule of all mediate inference so long as numbers or numerical proportions form no part of the data or premises.

(*b*) Again, so long as the middle term is singular both premises must be singular, and therefore the conclusion is usually singular.[1] But when the middle term

[1] For the middle term must be either subject or predicate in the premises. If it is subject and singular the proposition

is general, the premises may be general or particular as well as singular. And then the question arises as to what kind of a conclusion can be drawn, whether it can be universal or must be particular. Here applies obviously the general rule of formal inference already stated in an earlier chapter, namely, that the conclusion must not distribute a term which was not distributed in its premise. If, therefore, the term which becomes the subject of the conclusion (*the minor term,* as it is called) is not distributed in its premise (*the minor premise*) then the conclusion (if any) can only be particular.

(*c*) Once more, so long as a term is singular it is always distributed, no matter whether it is the subject or the predicate of a proposition, and no matter if it is predicate of an affirmative or of a negative proposition. But when the terms of a mediate argument are general, it is quite different. In such an affirmative proposition the predicate is never distributed; in a negative proposition it always is. Now if a conclusion is negative its predicate will be distributed; but it must not be distributed unless that term (*the major term,* it is called) is distributed in its premise (*the major premise*). This means that a negative conclusion is invalid unless the major term is distributed in the major premise.

(*d*) Again, when considering mediate inference with a singular middle term, it was shown that when both premises are really negative then no mediate inference is valid. This rule holds good also when the middle term is general. The reasons are the same.

is singular (by definition). If it is predicate and singular, it can only be predicated of a singular subject, and then the proposition is again singular.

(*e*) Similarly, in the case of mediate inference with a general middle term, as in the case of mediate inference with a singular middle term, and for the same reasons, when both premises are affirmative the conclusion can only be affirmative. And in this case we are not confined to the relation of identity, but can deal in the same way with all transitive relations.

(*f*) Lastly, if one premise is affirmative and the other is negative, only a negative conclusion can be inferred, when the middle term is general as when it is singular, and for the same reasons. And conversely, a mediate negative inference can only be drawn when one premise is affirmative and the other negative. This follows from (*d*) and (*e*) above. If two negative premises warrant no inference whatever, and two affirmative premises can only justify an affirmative conclusion, then a negative inference can only be drawn from two premises one of which is affirmative and the other negative.

§ 2. *General Rules of Mediate Inference.*

It will have been seen that mediate inference when the middle term is singular is simpler in various ways than when the middle term is general, because the whole question of the distribution of terms does not arise, singular terms being distributed in any case. At the same time the conditions concerning the distribution of terms when the middle term is general apply also to the case when the middle term is singular, even if they call for no special attention, because they are inevitably satisfied under the circumstances. The more comprehensive conditions (or rules or norms) of mediate inference when the middle term is general may, therefore, be formulated as the general rules of

mediate inference whether the middle term is general or singular. We may accordingly set out completely the general conditions or rules of mediate inference as follows:

1. There are three propositions—two premises and a conclusion.
2. There are three distinct terms, one of which (the middle term) occurs in both premises, each of the other two only in one of the premises, but also in the conclusion—the term which is the subject of the conclusion is called the minor term, and the term which is the predicate of the conclusion is called the major term.
3. The middle term must be distributed once at least.
4. No term may be distributed in the conclusion if it is not distributed in its premise—in other words, the conclusion may be universal only if the minor term is distributed in its premise, and negative only if the major term is distributed in its premise.
5. No conclusion may be inferred from two irreducibly negative premises—or, in other words, one premise at least must be affirmative.
6. If both premises are affirmative the conclusion can only be affirmative.
7. If one premise is negative the conclusion can only be negative, and conversely if the conclusion is to be negative one of the premises must be negative.

These general rules of mediate inference (or general rules of the syllogism,[1] as they are more usually called) could, of course, be reduced in number, if our

[1] This will be explained in the next chapter.

object were to formulate only principal rules, not derivative ones. In fact, we arrived at them without relying on much more than (1) the need of a real middle term, or mediating link, (2) the Law of Contradiction, and (3) the rule of all formal inference that the conclusion must not distribute a term undistributed in the premises. But the explicit formulation of all the important rules is much more helpful.

The foregoing general rules are quite sufficient to determine the nature or validity of a mediate inference in relation to given premises. What follows is only an elaboration of derivative details, which may be useful and interesting, but are not of the same importance. Once the general rules are mastered there should be no difficulty in the elaboration of the consequent details.

The three most comprehensive consequences which follow from the general rules of mediate inference are the following three corollaries :

(*a*) If both premises are particular, and do not specify certain numbers or proportions, no conclusion can be drawn.[1]

(*b*) If one premise is particular the conclusion (if any) can only be particular.

(*c*) If the major premise is particular, and the minor premise is negative, no conclusion can be inferred.

[1] Two particular premises such as

Most M's are P,
Most M's are S,

do warrant the conclusion

Some S's are P,

because the proportions of *M* are such as to ensure a real middle term, or common element, in the two premises.

The reason for all the corollaries is to be found in the fact that not enough terms are distributed, under the conditions supposed, to warrant anything else than is prescribed by the corollaries. Let us consider them each separately.

(*a*) Both premises are assumed to be particular. There are only two possibilities which need be considered: (i) Either both premises are affirmative, or (ii) one premise is affirmative, and the other is negative. If (i), then no term, not even the middle term, is distributed, so there can be no conclusion. If (ii), only one term will be distributed, namely, the predicate of the negative premise. But in this case at least two terms should be distributed to justify a conclusion, namely, the middle term and the major term. Therefore no conclusion.

(*b*) One premise is assumed to be particular. If there is to be a conclusion at all, the other premise must be universal (*a*). Now there are only two possibilities which need be considered. (i) Either both premises are affirmative, or (ii) one premise is affirmative, and the other is negative. If (i), then the two premises will only distribute one term between them, namely, the subject of the universal premise. Now to warrant a conclusion the distributed term must be the middle term, so the minor term is not distributed in its premise, and must not be distributed in the conclusion, that is, the conclusion can only be particular. If (ii), two terms will be distributed in the premises, namely, the subject of the universal premise, and the predicate of the negative premise. The inference in this case can only be negative, and so at least two terms must be distributed, namely, the middle term and the major term. The minor term, in that case,

will not be distributed, and the conclusion can only be particular.

(*c*) Here the minor premise is assumed to be negative, and the major premise particular. If, therefore, there is to be any chance of a valid inference, the major premise must be affirmative. But if the major premise is particular affirmative, it distributes neither of its terms. So the major term is not distributed. Since, however, one premise is negative, the conclusion, if any, will have to be negative, and distribute the major term. Therefore there can be no conclusion in this case.

CHAPTER X

DEDUCTION AND SYLLOGISM

§ 1. *Mediate, Deductive, and Syllogistic Inference.*

Deduction, or deductive inference, is usually defined as inference from a general proposition, or from general propositions, or as the application of laws (or rules) to relevant cases. It is usually contrasted with induction, which is inference from particulars. And syllogism is usually described at once as mediate inference and as deductive inference, and also as confined to propositions which express the relation of attribute to substance. If this conception were strictly adhered to, the term syllogism, would coincide with only those mediate inferences which have a general middle term, and which only deal with relations of identity and difference, or with relations which are reducible to those of identity or of difference. In practice, however, the treatment of the subject is not consistent in books on Logic, and some cases of mediate inference with a singular middle term, and without a general premise, are usually included among syllogisms. The reason for the inconsistency is to be found in an excessive leaning on the Aristotelian *dictum de omni et nullo* (or some similar formula) as the basis of syllogistic inference. Now, it is convenient to use the term *Deduction* in the above mentioned sense, which is different from that of *mediate inference*, in its widest sense; but there seems to be no good reason for using

the term *syllogism* in any other sense than that of mediate inference. After all, *syllogism* only means putting two and two together, and that is just what is done in *all* mediate inference. Except out of respect for tradition, the term syllogism might be dropped altogether, and be replaced by *mediate inference.* But there is no point in being unnecessarily iconoclastic so long as serious disadvantages, such as those of ambiguity, can be avoided. The term *Syllogism* also has the advantage of greater brevity than the term *mediate inference.* In this book, accordingly, the term *syllogism* will be understood as synonymous with *mediate inference*—a wider meaning than it has in other books on Logic. The term *Deduction* should also be used more consistently than is usually the case. Instead of confining it, as is nearly always done, to syllogistic inference with a general premise, it should be used to include also such inferences as immediate inference from an *A* to an *I* proposition, or from an *E* to an *O* proposition (usually called immediate inference by subalternation). According to the usage of terms here suggested and employed, an inference may be both deductive and mediate (or syllogistic), or it may be mediate without being deductive, or deductive without being mediate. For example, the argument $X = Y$, $Y = Z$, $\therefore X = Z$, is mediate, but not deductive; the argument, *All radii of the same circle are equal,* $\therefore$ *radii AB, AC, AD of circle BCD are equal,* is deductive, but not mediate; lastly, the argument *All who profit from war-preparations are prone to believe in the inevitableness of warfare. Manufacturers of arms and armaments profit from war-preparations,* $\therefore$ *they are prone to believe in the inevitableness of warfare,* is both deductive and mediate.

We shall now proceed to consider the varieties of mediate inference, or syllogism.

§ 2. *Figure and Mood of Syllogisms.*

Every syllogism, as we have seen, has two premises, a major premise and a minor premise. The major premise contains the middle term and the major term (*M* and *P*); the minor premise contains the middle term and the minor term (*M* and *S*). Now in each premise either of its terms might be subject and the other predicate. In other words the major premise might be either $M-P$, or $P-M$, and the minor premise might similarly be either $M-S$, or $S-M$. Consequently, there are four possibilities with regard to the arrangement of the terms in a syllogism:

Major premise:	$M—P$,	$P—M$,	$M—P$,	$P—M$.
Minor premise:	$S—M$,	$S—M$,	$M—S$,	$M—S$.

The conclusion in each case is assumed to be $S—P$, the names of the terms and of the premises being based on this assumption. Now these differences in the arrangements of the terms of a syllogism are called differences of *Figure*. The above four arrangements are obviously the only possible ones, and they are known respectively as the First, Second, Third, and Fourth Figure. The First Figure is the one most commonly used. It is the most natural Figure, as the term which is subject of the conclusion is also subject of its premise, and the term which is predicate in the conclusion is also predicate in its premise. In the Fourth Figure the arrangement of terms is just the opposite to that in the First Figure. In the Second Figure the middle term is predicate in both premises.

In the Third Figure the middle term is subject in both premises.

It will be seen that each Figure is a kind of scheme of syllogisms. With the same arrangement of terms in the premises, the premises may obviously vary in quality and quantity. These differences of quality and quantity were abstracted from, when differences of Figure were described. But, of course, there can be no proposition without some quality and quantity. Now differences in the quality and quantity of the propositions constituting a syllogism are called differences of *Mood*. And *prima facie* each Figure may have a number of different moods. For example, the syllogistic types

$$\begin{array}{lcl} MaP & \qquad & MeP \\ SaM & & SaM \\ \therefore SaP & & \therefore SeP \end{array}$$

are different moods of Figure I. The problem thus arises as to how many valid moods there are in each Figure, and what they are. This problem is known as that of the determination of the valid moods of the syllogism, and the problem is solved with the aid of the general rules of the syllogism and the corollaries already stated and explained.

§ 3. *The Determination of the Valid Moods.*

The general rules of the syllogism and the corollaries state what kind of premises, as regards quality and quantity, can or cannot justify a conclusion. Our best plan will therefore be, in the first instance, to abstract from differences of Figure, to consider the various conceivable combinations of premises having regard to their quality and quantity only, and to find

out which of them are disqualified, by the general rules and corollaries, from yielding a valid conclusion in any case, that is, in any Figure. Now as regards quality and quantity there are four propositional forms, *A*, *I*, *E*, and *O*. *Prima facie* any one of these might serve as major premise, or as minor premise, or both. There are consequently sixteen conceivable combinations of premises. Putting the major premise first, and the minor premise second, the sixteen possible combinations of premises are as follows:

AA, *AI*, *AE*, *AO*,
IA, *II*, *IE*, *IO*,
EA, *EI*, *EE*, *EO*,
OA, *OI*, *OE*, *OO*.

Of these sixteen combinations, eight cannot justify any conclusion. They are: *EE*, *EO*, *OE*, *OO* (Rule 5), *II*, *IO*, *OI* (Corollary (*a*)), and *IE* (Corollary (*c*)).

Thus only eight of the conceivable combinations of premises are likely to justify a conclusion in some Figure or other. They are *AA*, *AI*, *AE*, *AO*, *IA*, *EA*, *EI*, *OA*. But we cannot say what conclusion, or in which Figure, without examining each of the surviving combinations of premises in relation to each of the Figures. The reason for this should be obvious. Each combination of premises means something different in each figure, and consequently it may imply one kind of conclusion in one Figure, another in another, and perhaps no conclusion at all in a third. For example, *AA* means *MaP*, *SaM* in Fig. I and implies *SaP*; it means *PaM*, *SaM* in Fig. II and implies nothing at all here, because the middle term is not distributed. So we must examine each of the eight

surviving combinations of premises in each of the four Figures.

AA, AI, AE, AO, IA, EA, EI, OA.

FIGURE I. Scheme:

$$\begin{array}{l} M-P \\ S-M \\ \therefore S-P. \end{array}$$

Valid moods[1]: *AAA, AII, EAE, EIO.*

The premises *AE* and *AO* justify no conclusion because the conclusion (if any) could only be negative, and the major term is not distributed. *IA* and *OA* fail likewise because the middle term is not distributed.

[1] The procedure in determining the valid moods in each Figure may be briefly summarized as follows. Write down the premises in the form which they assume in the Figure under consideration (e.g. Fig. 1.

$$\begin{array}{l} MaP \\ SaM \end{array}$$

or Fig. II

$$\begin{array}{l} PaM \\ SaM \end{array}$$

and so on).

Then see if M is distributed in one of the premises. If not, then no conclusion follows.

If M is distributed, and if both premises are affirmative, then an affirmative conclusion follows, and the conclusion is universal if S is distributed in the minor premise, particular if it is not. But if one premise is negative, then not only must M be distributed, but also P. If both M and P are distributed in the premises, and only then, a negative conclusion follows, and the conclusion is universal if S is distributed in its premise, particular if it is not.

FIGURE II. Scheme:

$$\begin{array}{l} P-M \\ S-M \\ \therefore S-P. \end{array}$$

Valid moods: *AEE, AOO, EAE, EIO.*

The premises *AA, AI, IA* yielded no conclusion because the middle term is not distributed; and *OA* because the major term is not distributed, while the conclusion (if any) would have to be negative.

FIGURE III. Scheme:

$$\begin{array}{l} M-P \\ M-S \\ \therefore S-P. \end{array}$$

Valid moods: *AAI, AII, IAI, EAE, EIO, OAO.*

The premises *AE* and *AO* justify no conclusion because the major term is not distributed, while the conclusion (if any) could only be negative.

FIGURE IV. Scheme:

$$\begin{array}{l} P-M \\ M-S \end{array}$$

Valid moods: *AAI, AEE, IAI, EAO, EIO.*

The premises *AI* and *AO* yield no conclusion because the middle term is not distributed; and *OA* because the major term is not distributed, while the conclusion (if any) could only be negative.

§ 4. *Special Rules of Each Figure.*

If the valid moods of each Figure are examined in turn, it will be found that each Figure has its peculiarities and that these peculiarities are the logical result of the application of the general rules of the syllogism to the special arrangement of the terms in each Figure. These peculiarities constitute what are known as the *special rules of each Figure,* which may be considered now.

(*a*) *Special Rules of Figure I.* A glance at the valid moods of Figure I shows that

(i) the minor premise is always affirmative, and
(ii) the major premise is always universal.

The reason is that in this Figure ($M-P$, $S-M$, $\therefore S-P$) a negative minor premise, involving an affirmative major premise, would necessitate an undistributed major term (unless it happened to be singular) and so could not warrant a negative conclusion, which is the only one possible if one premise is negative. The minor premise must therefore be affirmative, and so does not distribute the middle term (unless it is singular), which must consequently be distributed in the major premise, and this can only be done if the major premise is universal.

(*b*) *Special Rules of Figure II.* It will be noticed that in all the valid moods of this Figure

(i) one premise is negative,
(ii) the major premise is universal.

The reason is as follows: In this Figure ($P-M$, $S-M$, $\therefore S-P$) the middle term is predicate in both premises, and (unless it is singular) can therefore

only be distributed if one premise is negative. Now a negative premise necessitates a negative conclusion (if any), and this requires the distribution of the major premise, which must consequently be universal.

(*c*) *Special Rules for Figure III.* In all the valid moods of this Figure

(i) the minor premise is affirmative,
(ii) the conclusion is particular.

The reason for (i) is the same as in the case of Figure I, namely, to prevent illicit distribution of the major term in the conclusion. But the minor premise being affirmative it cannot distribute the minor term in this Figure ($M-P$, $M-S$, $\therefore S-P$), unless it is a singular term. Therefore the conclusion cannot be general, only particular (or singular).

(*d*) *Special Rules of Figure IV.* The peculiarities of the moods of this Figure are not so obvious, but they will be seen to be the following:

(i) when either premise is negative the major is universal,
(ii) when the major premise is affirmative the minor is universal, and
(iii) when the minor premise is affirmative the conclusion is particular.

The reason for (i) is obvious if one looks at the scheme of this Figure ($P-M$, $M-S$, $\therefore S-P$). A negative premise necessitates a negative conclusion, if any, and this requires the distribution of the major term in the major premise, which must accordingly be universal (compare the special rules for Figure II). Again, (ii) if the major premise is affirmative it does not distribute the middle term (unless it is singular),

which must consequently be distributed in the minor premise, and so the minor premise must be universal. Lastly, the reason for (iii) is the same as in Figure III, namely, the minor term being undistributed in its affirmative premise (unless it is singular) must not be distributed in the conclusion.

It only remains to point out again that the special rules of the Figures are simply derived from the general rules of the syllogism, and involve no new principles. When testing the validity of any syllogism it is quite unnecessary to apply the special rules of the Figure to which it belongs, or indeed to trouble about its Figure at all; the general rules of the syllogism can be applied to it directly.

CHAPTER XI

ABRIDGED SYLLOGISMS AND CHAINS OF SYLLOGISMS

§ 1. *The Order of Propositions in the Syllogism as a Common Form of Argument.*

Syllogisms are very common forms of argument or reasoning. Their frequency is obscured by certain facts which we have to consider next.

In the first place, as a matter of convenience in the exposition of the different types of syllogisms, they are usually arranged, in books on Logic, in a certain order—major premise first, minor premise next, and conclusion last. There is nothing sacred about this order, any other order would do equally well; only it saves time to have some uniform arrangement for purposes of exposition, because the character of the several constituent propositions can be recognized at a glance. In actual argument, the arrangement varies in all possible ways. Perhaps the commonest arrangement begins with the conclusion, so as to make clear at once what one is driving at. This is nearly always the case when there is a reasoned answer to a question. But whatever the sequence of propositions in actual argument there should be no difficulty in distinguishing the premises from the conclusion, or the major premise from the minor premise, though even this requires some practice in the analysis of arguments.

Let us consider an illustration or two. Take the argument, "Triangles inscribed in a circle with the diameter for base and the vertex on the circumference have the sum of the squares on their two sides equal to the square on their base. For such triangles are right-angled at the vertex; and all right-angled triangles have the sum of the squares on the sides subtending the right angle equal to the square on the base (or hypotenuse)." Here the order of propositions is, conclusion, minor premise, major premise. If re-arranged, for purposes of easy scrutiny, in the conventional order adopted in this book, it would fall into the following scheme:

$$\begin{array}{l} MaP \\ SaM \\ \therefore SaP. \end{array}$$

Take another example. Suppose a teacher asks what part of speech the word *planet* is, and why? The pupil would probably say: "*Planet* is a common noun, because it is a word which can be applied in the same sense to any one of a certain class of things, and such words are common nouns." Here, likewise, the reasoned answer begins with the conclusion, goes on to the minor premise, and ends with the major premise. In logical form and character it is the same as the preceding syllogism.

§ 2. *The Abridgment of Syllogisms and the Universe of Discourse.*

In the second place, owing to the laudable tendency to be brief, and to put some trust in the intelligence of our fellows, syllogisms are usually abridged, in

actual discourse, by the omission of one or other of the constituent propositions, though these are obviously implied, or taken for granted. To insist on each syllogism being expressed completely on every occasion would be as absurd as the pedantry of a nursery governess who insists on complete sentences. An abridged syllogism is called an *enthymeme*, and is said to be of the first, second, or third order according as the omitted proposition is the major premise, the minor premise, or the conclusion.

In conversation, and in discussion generally, there is a wise tendency to be brief, by omitting whatever is obvious to an intelligent person. No doubt there are occasions when it is safest to be as explicit as possible. That is why, for instance, a lawyer's brief never is brief. But in ordinary intercourse there is as little inclination to regard legal documents as a model of self-expression as there is to regard legal process as a model of social etiquette. What happens for the most part is this. There is a mutual understanding about the general topic, or sphere of reference, which is under discussion. This sphere of reference is known as *the universe of discourse* or *limited universe* [1] (or *suppositio*, in Latin). And the fact that the conversation or discussion is understood (informally, of course) to be limited to a certain range of topics, instead of being directed to the world at large, enables the speakers to be less explicit, and consequently more brief, than would otherwise be the case. Thus, for example, if the general theme under discussion is

[1] A full account of the conception of *universe of discourse* will be found in the author's *Studies in Logic* (Cambridge University Press), Chap. III.

modern literature and somebody remarks that "Machiavelli is rather long-winded," it will be understood that the reference is not to the historical person, but to the novel. In fact "Machiavelli" stands for "the novel Machiavelli," just as, when speaking of portraits, "Gladstone" would stand for "the portrait of Gladstone," and so forth. Of course, there are people who insist on taking us too literally, just as there are others who will leave nothing to our imagination or to our understanding; we must accept them with resignation, like other trials, if we cannot escape them. Now sometimes whole arguments are abridged, thanks to mutual understanding, just as at other times single sentences or phrases are. In such cases the real argument as a whole cannot be duly evaluated unless all the omissions are made good, and taken into account. Sometimes an argument is really much stronger than appears at first sight when the omitted but assumed links have not yet been interpolated. But at other times an argument in its abridged state may appear much stronger than it is when completed, because some of the suppressed premises may appear more than doubtful when stated explicitly. In some cases the abridgment of an argument is due not so much to want of time as to lack of candour. Now it may or may not be frequently necessary to examine the validity of one's own or other people's arguments; but when it is necessary to do so, then the whole argument must be set out as explicitly as possible, by supplying all the assumed but suppressed propositions. When the argument consists of a single abridged syllogism this is not a difficult matter, and expertness with single syllogisms enables one to cope readily with chains of abridged syllogisms.

And now for a few simple illustrations of abridged syllogisms (or enthymemes). The examples given in § 1 may serve our purpose. In an actual geometry class, in which the Pythagorean theorem had just been dealt with, the argument about triangles inscribed in a circle with its diameter for base would ordinarily be abridged by the omission of the last proposition, which is the major premise. If, on the other hand, what had most recently been explained was that such triangles are right-angled, the minor premise would probably be omitted as something obvious, while the major would be stated. Similarly with the grammar question. If the children had just been taught the definition of a common noun, the answer would probably omit the major premise; if not, it might omit the minor premise. Sometimes, again, the premises are given, but the conclusion is not stated. This is usually the case with suggestions, and with insinuations, especially with unpleasant insinuations. For example, "People who trade honestly don't get rich so quickly as Mr. X. did." This is equivalent to: *No people who work honestly get rich so quickly, Mr. X. is a person who did get rich so quickly [Therefore Mr. X. is not honest].*[1]

$$MeP$$
$$SaM$$
$$[\therefore SeP].$$

Lastly, the following instance may show, in a simple way, how easy it is to mislead one by suppressing an

[1] Brackets are used to indicate that the proposition enclosed in them was omitted from the wording of the original argument, but assumed by it.

inconclusive premise. If someone says "Miss Smith cannot yet be thirty years of age, because she has no Parliamentary vote," probably few people would suspect its accuracy; but if the major premise, perhaps obscurely in the speaker's mind, were stated explicitly, namely, "All women without a vote are under thirty," most people would recognize its inaccuracy. In the example just given the conclusion followed from the premises, only the major premise, was false—the form of the syllogism being

$$\begin{array}{c} [MaP] \\ SaM \\ \therefore SaP. \end{array}$$

Sometimes, however, the assumed but suppressed premise is true, only it does not support the inference drawn. In the foregoing illustration, for instance, the premise in the speaker's mind may have been "All women voters are thirty years of age or over." In that case the form of the syllogism would be

$$\begin{array}{l} [MaP] \\ SeM \\ \therefore SeP\text{—an incorrect conclusion.} \end{array}$$

§ 3. *Chains of Syllogisms and of Abridged Syllogisms.*

In the third place, it is not very often that a problem can be settled with the aid of one syllogism, though such cases are by no means uncommon. For the most part arguments involve several interconnected syllogisms (or chains of syllogisms, or *polysyllogisms*, as they are called)—to say nothing of other forms of inference—and as each syllogism tends to be abridged,

in the way just described, what we commonly find are chains of abridged syllogisms. Naturally, such arguments are not readily recognized, or easily deciphered, by those who have only a very superficial acquaintance with Logic, and have never made a serious attempt to study arguments in the flesh, so to say.

A *chain* of syllogisms, as distinguished from a mere group of disconnected syllogisms, is characterized by the fact that each syllogism either supports, or is supported by, the other—there is real connection, or dependence, between them. A supporting syllogism is usually called, in relation to the supported or dependent syllogism, a *prosyllogism*; the dependent syllogism, in relation to its *prosyllogism*, is called an *episyllogism*. The prefixes *pro-* and *epi-* mean, of course, *before* and *after*, respectively; but in this case the reference is not to sequence in time, but to *logical* sequence—the conclusion of the prosyllogism is used as a premise of the episyllogism, which is therefore logically dependent on the former, whichever syllogism may be stated first. Take, for instance, the following argument: *Large-scale production means an increase in real income, for it cheapens articles of ordinary consumption, and so makes our incomes go farther than they would otherwise. Now an increase in real income is obviously a boon to people of small means. Therefore large-scale production is a boon to people of small means.*" Here we have two syllogisms. In the first the order of statement is that of conclusion first, minor premise next, major premise last. In the second syllogism the major premise is stated first, and is followed at once by the conclusion—the minor premise is supplied by the conclusion of the first syllogism, and is therefore

not repeated. Set out in the conventional order the argument has the following form:

MaP
SaM *Prosyllogism.*
∴ *SaP*
PaQ
[*SaP*] *Episyllogism.*
∴ *SaQ*

Now the actual order of statement might just as well have been reversed: *Large-scale production is a boon to people of small means, because it means an increase in their real incomes, which is a boon. For large-scale production cheapens articles of ordinary consumption, and so makes our incomes go farther than they would otherwise.* Symbolically it runs:

SaQ
SaP *Episyllogism.*

[*SaP*]
∵ *SaM* *Prosyllogism.*
MaP

Here, though the order of statement is reversed, the supporting syllogism being stated after the supported syllogism, yet logically the supporting syllogism remains the prior or *pro*-syllogism. If an argument is arranged in the first of these ways, so that it passes from prosyllogism to episyllogism, that is, always towards an episyllogism, it is said to be episyllogistic, or progressive, or synthetic. If, on the other hand, it is expressed in the second way, passing from episyllogism to prosyllogism, that is, always towards a prosyllogism, it

is said to be prosyllogistic, regressive, or analytic. A progressive chain of abridged syllogisms is called a Sorites; a regressive chain of abridged syllogism is called an Epicheirema. But usage is not uniform in these matters, and there is no particular virtue in these technical expressions. The main thing is to be able to recognize the real character of such chains of syllogisms, to analyse them into the constituent single syllogisms, and note their conformity or otherwise to the general rules of the syllogism.

§ 4. *Degrees of Complexity, or Linear and Systematic Inference.*

Chains of syllogisms and of abridged syllogisms may obviously be of all degrees of complexity. The above example was of the simplest possible type. Here is a symbolic illustration of a more complex type, which would not be difficult to match in *Euclid* or elsewhere.

$$\begin{array}{llllll}
 & MaP & \because [NaP], & MaN & \because ZaN, & [MaZ] \\
 & SaM & \because RaM, & [SaR] & & \\
\therefore & SaP & & & & \\
 & PaQ & \because [VaQ], & PaV & \because [TaV], & PaT \\
 & [SaP] & & & & \\
\therefore & SaQ & & & &
\end{array}$$

In the simplest types of polysyllogisms, or sorites, the final inference is arrived at in a comparatively simple and straightforward manner, and the symbolic arrangement is in a straight vertical line; but as it gets more and more complicated, the argument, even when set out symbolically, assumes the appearance of a more and more complicated figure, or complex interconnected system. Hence we may distinguish

between the comparatively simpler *linear* inference, and the more complex *systematic* inference. But it would be an obvious blunder to identify syllogistic inference with linear inference.

The degree of complexity of an argument varies partly with the extent of the thinker's knowledge, or beliefs, about the various topics relevant to the problem, and partly with the extent to which his views on these relevant topics are shared by those to whom the argument is addressed, if it is addressed to others. The larger the number of connected topics on which the thinker already has definite views, the simpler will be the process of inference by which he endeavours to solve his immediate problem. Similarly the greater the agreement between the thinker and his audience on connected topics, the simpler will the argument be, and the greater the differences, the longer and the more comprehensive the discussion. That is why an argument which is sufficient for the followers of one party or school of thought rarely satisfies those of another. If the difference extends to fundamental principles, then the argument is apt to become metaphysical and—interminable.

CHAPTER XII

HYPOTHETICAL PROPOSITIONS AND INFERENCES

§ 1. *Categorical and Hypothetical Propositions.*

The objects of our thought vary enormously in respect of concreteness (or, its opposite, abstractness). Sometimes we think of and about some particular object or situation—some individual person, some domestic pet, some particular planet, and so on. Sometimes we think about types of objects, rather than individual members as such—human beings generally, horses or dogs generally, planets or stars in general. At other times the objects of our thought are even less concrete, or more abstract, we may think about general laws of human development, or about the general anatomical structure of quadrupeds, or the general conditions and laws of their evolution, or about the laws of planetary motion, the law of gravitation, and so on. The more general considerations are more abstract in the sense that they concern certain characteristics or properties and their mutual relations without special regard to their particular embodiments or settings.

The term *concrete* is derived from the Latin *concrescere* (to grow together), and anything which is thought of as a centre, or a coalescence of many attributes and relations is said to be regarded as concrete, and the

more concrete, the greater the number of attributes and relations associated with it. On the other hand, when the attributes or relations are thought of more or less apart from the things or situations which they characterize, they are said to be regarded abstractly. There are various degrees of abstraction—a general term like *Englishman* is more abstract (or less concrete) than a singular term like *John Locke,* and a more general term like *man* is more abstract (or less concrete) than the term *Englishman,* while the name of a quality, like colour, or a relation, like *partnership,* is more abstract still. Nowadays only terms like the last two examples are usually called *abstract terms,* and general class names like *man* are called concrete; but not so long ago such general names were also called abstract names, because of the abstraction from the varying qualities, etc., of individuals and sub-classes which they involve.

Now propositions vary considerably in certain respects according as their terms are concrete or abstract. From the point of view of human knowledge it is obvious that the amount of knowledge embodied in a singular proposition (that is a proposition having a singular term for subject) cannot as a rule be compared in extent with that embodied in a general proposition; and, as will appear soon, the higher achievements of science are embodied in highly abstract propositions.

Again, one may assert something of a concrete subject without knowing in virtue of which attributes of the subject the predicate belongs to it. Such assertions are sometimes described as assertions of brute fact, in contrast with assertions of real connections between conditions and results. This distinction, and the other distinction just explained, are intimately

connected. Assertions of brute fact are naturally expressed in the more concrete type of proposition; assertions of connection between conditions and results (in the widest sense of these terms) are as naturally expressed in the more abstract types of proposition.

Broadly speaking, the categorical type of proposition is the natural vehicle of the more concrete assertions of brute fact; the hypothetical proposition is the natural vehicle of the more abstract assertions of universal connections. It must not be forgotten, however, that in actual use the distinction between the two types of proposition is sometimes effaced, more or less abstract assertions of connection being expressed in the categorical form of proposition, while concrete assertions of brute fact, or of arbitrary stipulation, are expressed in the hypothetical form of proposition.

§ 2. *The Meaning and Implications of the Hypothetical Proposition.*

In the hypothetical form of proposition we have not merely a subject and a predicate, but an antecedent and a consequent, or a condition and result, between which a connection is asserted. Symbolically its general form is

If A, then C,

where *A* represents an antecedent, or condition, and *C* a consequent or result. For example, *If a triangle is equilateral, it is equiangular; If the temperature of a gas is raised, its volume is increased* (the pressure being constant); *If the supply of a commodity falls short of the demand, the price tends to rise* (in the absence of Government control). In all these cases there is an

assertion of a general connection between an antecedent and a consequent, without special reference to actual or concrete cases. Of course, the assertion has application to concrete cases. In one way or another, assertions are usually grounded in reality, and have reference to the world of reality. Life is too short and strenuous to be spent on unrealities. In one very real sense, the very abstractness of abstract assertions is only intended to give them a wider applicability, that is, to make them applicable to a wider range of facts. For that reason it is always possible to restate the more abstract assertions in more concrete form, to reduce the hypothetical form *If A, then C* to the categorical form *Every case of A is a case of C.* Thus the above examples of hypothetical propositions can be expressed as categorical propositions in the following way: *Equilateral triangles are equiangular; A gas, the temperature of which is raised, increases in volume; A commodity the supply of which falls short of the demand tends to rise in price.*

At the same time there are such things as suppositions which, though based on facts, may still be merely speculative, and may not refer to any actual instances. One might, for example, in the light of what is known about the effects of variations in temperature on all kinds of substances, think of what might happen to various substances if reduced to a temperature of absolute zero, although no actual instance of that temperature is known. Such suppositions, if they are to be expressed unambiguously, should be expressed only in the hypothetical form, and not in the categorical form, which would probably mislead people into supposing that the reference is to actual instances of the phenomenon in question.

The main feature, then, of the hypothetical type of proposition is the assertion of a connection between an antecedent (or a condition) and a consequent (or a result). Now the nature of the connection between the antecedent and the consequent must be grasped clearly, if misinterpretation and invalid inferences are to be avoided. In all cases in which the form of assertion *If A, then C* is employed intelligently, it means that the antecedent (*A*) cannot be true without the consequent (*C*) being true likewise. We have just stated that, expressed categorically, it means *AaC* (*every case of A is a case of C*); and to suppose that in some cases the antecedent might also be true although the consequent is not true, would be equivalent to asserting *AoC*, its contradictory. The consequent, then, must be admitted to be true, whenever the antecedent is true, if we commit ourselves to an hypothetical assertion; so that if in any case, or cases, the consequent is not true, then the antecedent cannot be true either. We can express this by saying that *If A, then C* implies *If not C, then not A*. On the other hand, though *A* cannot be true without *C*, yet *C* may be true without *A*. *If A, then C*, in other words, does *not* imply *If C, then A*. This is clear if we express them in categorical form. *If A, then C* means *AaC*; *If C, then A* means *CaA*. But we have seen that universal affirmative proposition may not be converted simply. *AaC* does *not* imply *CaA*, only *CiA*, that is, *some cases in which the consequent is true are cases in which the antecedent is true*, which is not the equivalent of *If C, then A*. To treat *If A, then C* as though it implied *If C, then A* is essentially the same mistake as to convert *SaP* into *PaS*, though the fallacies have different names, namely, *the fallacy of consequens*, and *the fallacy*

of illicit conversion, respectively. Since, then, the consequent (C) may be true even when the antecedent (A) is not, it follows that one may not infer the falsity of the consequent from the falsity of the antecedent—in other words *If A, then C* does *not* imply *If not A, then not C*. There are, indeed, some cases in which *If A, then C*, and *If C, then A*, and *If not A, then not C* are all true. This happens whenever the antecedent and the consequent are *reciprocal*, that is, each implies the other, as, for instance, in the case of *If a triangle is equilateral it is equiangular*, where it is also true that *If a triangle is equiangular it is equilateral.* But such reciprocity is not usual, and must not be assumed without special evidence. Where the reciprocal relation does hold good one is, of course, entitled to express it in the two propositions *If A, then C* and *If C, then A*. In such cases the proposition *If C, then A* is not implied by *If A, then C* but is an independent assertion, and the proposition *If not A, then not C* is not implied by *If A, then C* but by *If C, then A*.

Briefly, then, the hypothetical form *If A, then C* implies (i) that in every case in which A is true C must be true, and (ii) that in every case in which C is not true A cannot be true (*If not C, then not A*). But it does *not* imply (iii) *If C, then A*, nor (iv) *If not A, then not C*.

It will have been noticed that the antecedent and the consequent of an hypothetical proposition consist each of a categorical proposition of the form *S is P* or *S is not P*. The antecedent and the consequent may be either affirmative or negative, and the symbols A and C must not be regarded as standing for affirmations only. Just as x, y, z, etc., may stand for negative as well as for positive quantities, just as

S, *M*, *P*, etc., may stand for negative as well as for positive terms, so *A*, *C*, etc., may stand for a negative as well as for an affirmative antecedent, or consequent. Conversely, just as $-x$ may be a positive quantity, and $\bar{S}$ a positive term, so *not* $-$ *A* (or $\bar{A}$) and *not* $-$ *C* (or $\bar{C}$) may stand for an affirmative antecedent or consequent. Usually an hypothetical proposition with a negative consequent is regarded as a negative proposition. It certainly can be so regarded and treated. But if the foregoing remarks are borne in mind the whole business of formal inference from hypothetical premises is considerably simplified. It becomes possible to treat all hypothetical propositions as affirmative, even if their antecedents or consequents happen to be negative. This is legitimate, because, after all, the main function of an hypothetical proposition is to *affirm* a connection between the antecedent and the consequent; and the connection is *affirmed* just as much when the antecedent and the consequent are (either or both) negative as when they are affirmative. Thus, for instance, the assertion *If a triangle is not equilateral it is not equiangular* is of the type *If A, then C* just as much as the assertion *If a triangle is equilateral it is equiangular*, though, naturally, if the former is symbolized as it is symbolized here, the latter, if it occurs in the same argument, will have to be symbolized *If* $\bar{A}$, *then* $\bar{C}$. The form remains the same, only the terms (the antecedents and consequents) are different. We can, accordingly, treat all hypothetical propositions as affirmative. The denial of an hypothetical takes the form of asserting another affirmative hypothetical proposition having the same antecedent but a contradictory consequent. The contrary will be universal; the contradictory will be either particular

or modal.[1] Thus, for example, the contrary of *If A, then C* will be *If A, then* $\bar{C}$; its contradictory will be *If A, then sometimes* $\bar{C}$ or *Sometimes if A, then* $\bar{C}$, or *If A, then maybe* $\bar{C}$, or *If A, then not necessarily C.*

This brings us to the question of *particular* hypothetical propositions. Strictly speaking hypothetical propositions, when properly used, should only be universal, not particular, because if there is a real connection between the antecedent and the consequent, the proposition is universal; and if there is no such general connection, the assertion should be expressed in the categorical form, not in the hypothetical form. It will be found that in a particular hypothetical proposition the particle "if" really means "when." The difference between *if* and *when* is this: *if* introduces a *condition* of a certain event, etc., and a real condition is universal, and is best expressed in an hypothetical proposition; *when* introduces *instances* of a certain event, etc., and an assertion about instances, as already explained, is best expressed in the categorical form of proposition. The fact remains, however, that people do sometimes use particular hypothetical propositions, which have to be dealt with accordingly.

§ 3. *Pure Hypothetical Syllogisms.*

In the light of the foregoing explanations there should be no real difficulty in dealing with the immediate inferences from hypothetical propositions, or with mediate inferences from two hypothetical premises. In either case the hypothetical expressions can, if

[1] Propositions of the form *S may be P, S need not be P, S must be P, S cannot be P,* are called modal.

necessary, be expressed in categorical form, and then treated in the way already explained in connection with categorical propositions and inferences. An example or two may be given here of syllogisms with two hypothetical premises, or pure hypothetical syllogisms, as they are called.

If the rays of light coming from the fixed stars are subject to gravitation they will be bent by planets near their path to the earth ;
If the rays of light, etc., are material they are subject to gravitation ;
∴ *If the rays of light, etc., are material they will be bent by planets near their path to the earth.*

The form of this syllogism obviously is :

$$\left.\begin{array}{r} \textit{If } B, \textit{ then } C, \\ \textit{If } A, \textit{ then } B, \\ \therefore \textit{If } A, \textit{ then } C, \end{array}\right\} \begin{array}{c} \text{and} \\ \text{corresponds} \\ \text{to} \end{array} \left\{\begin{array}{l} MaP \\ SaM \\ \therefore SaP. \end{array}\right.$$

Here is an example with a negative consequent :

If a triangle is equiangular it is not right-angled ;
If a triangle is equilateral it is equiangular ;
∴ *If a triangle is equilateral it is not right-angled.*

This can be regarded as having the same form as the preceding example, but with a negative consequent in the major premise. Or it can be treated as having a negative major premise, in which case its form will be as follows :

$$\begin{array}{l} \textit{If } B, \textit{ then not } C, \\ \textit{If } A, \textit{ then } B, \\ \therefore \textit{If } A, \textit{ then not } C. \end{array} \quad \begin{array}{c} \text{corresponding} \\ \text{to} \end{array} \left\{\begin{array}{l} MeP \\ SaM \\ \therefore SeP. \end{array}\right.$$

Of course, C and P in the two cases have contradictory meanings.

Finally, an example of an invalid syllogism :

If a man is guilty he is uncomfortable under cross-examination ;
If a man is nervous he is uncomfortable under cross-examination ;
∴ *If a man is nervous he is guilty.*

The form of this syllogism is :

$$\left.\begin{array}{r} \textit{If } C, \textit{ then } B, \\ \textit{If } A, \textit{ then } B, \\ \therefore \textit{If } A, \textit{ then } C, \end{array}\right\} \begin{array}{c}\text{and}\\ \text{corresponds}\\ \text{to}\end{array} \left\{\begin{array}{l} PaM \\ SaM \\ \therefore SaP. \end{array}\right. .$$

This involves the fallacy of undistributed middle term.

§ 4. *Mixed Hypothetical Syllogisms.*

In addition to the mediate inferences which may be drawn from two hypothetical premises, it is also possible to draw mediate inferences from an hypothetical major premise and a categorical minor premise. Such mediate arguments are known as *mixed hypothetical syllogisms,* or hypothetico-categorical syllogisms. After what has already been said above about the meaning and implications of the hypothetical type of proposition very little need be added to explain the mixed hypothetical syllogisms. With a major premise of the form *If A, then C* there are only two ways in which a categorical minor premise can mediate an inference. The categorical minor must either (i) posit the antecedent (A), or (ii) reject the consequent (C). In the former case (i), the consequent is

accepted as conclusion; in the latter case (ii), the conclusion denies the antecedent. So we obtain two main types of mixed hypothetical syllogism, which are known respectively as *Constructive* and *Destructive*, and may be symbolized as follows:

Mixed Hypothetical Syllogisms

(i) *Constructive*	(ii) *Destructive*
If A, then C	*If A, then C*
	not C (*or* $\overline{C}$)
C	∴ *not A* (*or* $\overline{A}$)

For example:

(i) *If rays of light are material, they are subject to gravitation,*
Rays of light are material,
∴ *Rays of light are subject to gravitation.*

(ii) *If the rings of Saturn were non-material they would be invisible,*
The rings of Saturn are visible,
∴ *The rings of Saturn are material.*

It makes no fundamental difference to the form of the mixed hypothetical syllogism whether the antecedent and consequent are (both or either) affirmative or negative. But it should be remembered, of course, that the positing of a negative antecedent, or consequent, will give a negative minor premise, or conclusion, and the rejection of a negative consequent, or antecedent, will give an affirmative minor premise, or conclusion. So that the constructive or destructive character of a mixed hypothetical syllogism does *not* depend on the quality of the minor premise, or

clusion, but on their relation to the antecedent and the consequent of the major premise. A syllogism may be destructive even if its minor premise, or conclusion, is affirmative, and it may be constructive although the minor premise, or conclusion, is negative. The first of these points is illustrated by the preceding example (ii); the second point is illustrated by the following argument:

If carbon is not metallic it is not capable of powerful magnetic influence;
Carbon is not metallic;
∴ *Carbon is not capable of powerful magnetic influence.*

§ 5. *Abridged and Concatenated Hypothetical Syllogism.*

The remarks made in Chapter XI about abridged categorical syllogisms and chains of categorical syllogisms apply also, *mutatis mutandis*, to hypothetical syllogisms, pure and mixed. No special treatment is consequently necessary.

CHAPTER XIII

ALTERNATIVE (OR DISJUNCTIVE) PROPOSITIONS AND INFERENCES

§ 1. *The Meaning and Implications of the Alternative Proposition.*

Having considered categorical and hypothetical propositions, their meaning and their implications, we must consider next the disjunctive, or alternative, type of proposition. The essence of what is commonly called a disjunctive proposition is that it asserts that one or other of certain alternatives holds good. Its symbolic form may be best expressed thus : *Either* A_1, *or* A_2, where A_1 and A_2 stand for categorical propositions, such as *S is M, S is P, P is Q, S is not M*, etc. Just as the hypothetical proposition asserts a connection between A and C, and says in effect that the truth of A involves the truth of C, so the alternative proposition asserts that one of the alternatives (A_1 or A_2) is true, that both are not false.

To understand clearly the meaning and implication of the alternative type of proposition, it is necessary to realize how it comes to be used. Sometimes it arises out of the classification of things, qualities, etc. into classes and sub-classes. Lines, for example, are divided into the two sub-classes, right lines and curves. We consequently say *Lines are either straight or curved*, which means little more than *Some lines are straight, and*

some are curved. Similarly, British subjects consist of two sub-classes, namely, British-born, and naturalized. So we say *British subjects are either British-born or naturalized*, which, again, means little more than that *Some British subjects are British-born, and some are naturalized.* Of course there may be, and there frequently are, more than two sub-classes. For example, *Triangles are equilateral, or isosceles, or scalene*, which means little more than that *Some triangles are equilateral, some are isosceles, and some are scalene.* Now such propositions, though alternative in form, are really, as the second statement in each case shows, categorical propositions, or combinations of categorical propositions. But suppose now that we have to assert something about a subject, say S, which we know to be included in a certain class, say P, which has certain sub-classes, say p_1 and p_2. Now we may simply assert categorically *S is P*, which we could assert even if we did not know the sub-classes of P. But if we want to utilize our knowledge of the sub-classes of P we shall naturally assert *S is either p_1 or p_2.* For example, we may assert categorically that *Mr. X is a British subject*, or we may assert that *Mr. X is either a British-born or a naturalized British subject.* We know that one of the alternatives must be true, and if we should discover subsequently that, say, the first alternative is not true, then we shall know that the second is true. Now, if, as is usual, we call the wider class the *genus* of its sub-classes, and the sub-classes the *species* of their *genus*, then we can say that sometimes alternative propositions express our knowledge of the *generic* character of a subject, coupled with uncertainty about its *specific* character, though we know exhaustively what the various specific

characters are. In such cases, namely, those based on our knowledge of the classification of the relevant objects, the alternatives are not only exhaustive, so that one of them must be true, but (since, in a sound classification, the sub-classes are always mutually exclusive, as well as collectively exhaustive) they are also mutually exclusive, so that *only* one of them can be true. In other cases, however, as we shall see, the alternatives are not mutually exclusive, so that while one of them must be true, both may be.

There are cases, namely, in which the same kind of result can be achieved in various ways. The results achieved by such different means or methods are never precisely the same, but for some practical purposes they may be sufficiently similar to be regarded as essentially the same. Now suppose we know that there are only a certain number of ways, or means, say two (for the sake of simplicity), by which a certain kind of result can be obtained, then, although we cannot say categorically that it was, or will be, brought about in one way, we can still assert that one or other of these ways was, or will be, responsible for that result. For example, suppose we know that there are only two ways in which variations in the volume of a gas can be effected, namely, by changing its temperature, and by varying its pressure. In that case even if we do not know how exactly, in a particular instance, the change in the volume of a gas was effected, we can still assert, for instance, that *the increase in the volume of this gas is due either to an increase in temperature or to a decrease in pressure*. Similarly, if it is known that there are only two ways of achieving exceptional academic success, say, by exceptional cleverness and

exceptional industry, then, in the absence of other information, it can be asserted of any relevant person that *he is either exceptionally clever or exceptionally industrious.* Now in these and similar cases the alternative proposition still asserts the truth of one of the alternatives mentioned, but the alternatives are not mutually exclusive, so that although one alternative must be true, both may be true. In the above illustrations, for instance, the increase in the volume of the gas in question may be due to *both*, an increase in temperature and decrease in pressure; and the academic success of the person in question may be due to *both*, cleverness and industry. It will be seen, therefore, that it would be a mistake so to interpret the alternative form of proposition as to make the alternatives always mutually exclusive. The term *disjunctive*, which is commonly applied to alternative propositions, rather implies, or, at least, suggests strongly, that the alternatives are always mutually exclusive. It is, therefore, preferable to substitute the term *alternative*, which just brings out the fact that the propositions in question assert alternatives, without suggesting that they are mutually exclusive. In a great many cases the alternatives are as a matter of fact mutually exclusive. With a little care it is even possible so to express alternative propositions as to make the alternatives always mutually exclusive. For example, the proposition about the increase in the volume of the gas could be expressed in this way: *The increase in the volume of this gas is due to* (1) *an increase in temperature only, or* (2) *a decrease in pressure only, or* (3) *to both an increase in temperature and a decrease in pressure.* Or, more generally, the proposition *S is either P or Q* can be *restated* in the form

S is P only, or Q only, or both P and Q = *S is* $P\overline{Q}$ *or* $\overline{P}Q$ *or PQ*, where the various possibilities are mutually exclusive. But the cumbrousness and pedantry of such restatement are rather against it, and, in any case, the fact remains that people do employ the alternative form of proposition even when the alternatives are not mutually exclusive. The simplest way, accordingly, is not to regard the alternatives as mutually exclusive unless we have special reasons for it, that is, reasons other than the mere form of alternative assertion. We may conclude, therefore, that the alternative type of proposition asserts that one of the alternatives is true, and that it does *not*, as such, assert that *only* one of them can be true. From this it follows that the alternative proposition *Either* A_1 *or* A_2 implies *If not* A_1, *then* A_2 and *If not* A_2, *then* A_1 ; and that it does *not* imply *If* A_1, *then not* A_2 and *If* A_2, *then not* A_1. It is worth noting that in alternative propositions no real significance attaches to the order in which the alternatives are stated, so that, in the foregoing statement of what *Either* A_1 *or* A_2 implies and does not imply, the second proposition in each case was really unnecessary.

Alternative propositions are essentially affirmative. Whatever the alternatives may be, the proposition always asserts, in effect, that one of the alternatives *is* true. But the alternatives themselves (like the antecedents and consequents of hypothetical propositions) may be either affirmative or negative. Qualitatively, therefore, alternative propositions are always affirmative. The contradictory of *Either* A_1 *or* A_2 is *Neither* A_1 *nor* A_2. But this last proposition is not an alternative proposition, only a compound categorical proposition, meaning A_1 *is not true* (or *not* A_1)

and A_2 is not true (or *not A_2*)—there are no *alternatives* asserted.

Quantitatively an alternative proposition should always be universal, never particular. The use of a particular alternative proposition, such as *Sometimes either A_1 or A_2*, or *Some S's are either P or Q*, is simply a confession of ignorance of the other possibilities. The assertion in question would be expressed more accurately in two particular categorical propositions—*Some S's are P* and *Some S's are Q*. This should be obvious from such instances as *Some triangles are either equilateral or isosceles, Some heavenly bodies are either planets or comets.*

§ 2. *Pure Disjunctive Syllogisms.*

Having explained the meaning and immediate implications of the alternative type of proposition we may consider next the mediate inferences which may be drawn from two alternative propositions. The form of the mediating element (corresponding to the middle term of a pure categorical syllogism) is peculiar in this case. It should be obvious that two alternative premises, even with a common alternative, can yield no mediate conclusion. *Either A_1 or A_2*, and *Either A_2 or A_3* can only be summarized in the proposition *A_1 or A_2 or A_3*. Similarly, *S is either P or Q* and *S is either Q or R* can only be summarized in the proposition *S is P or Q or R*. But then a bare summary is not a mediate inference. There is no term here merely mediating between others and forming no part of the result, as in the case of pure categorical and pure hypothetical syllogisms. The only way in which mediate inference is possible with two alternative

premises is when an alternative of one premise contradicts an alternative of the other premise. Thus:

$$\left.\begin{array}{l} \textit{Either } A_1 \textit{ or } A_2 \\ \textit{Either not } A_2 \textit{ or } A_3 \\ \therefore \textit{Either } A_1 \textit{ or } A_3 \end{array}\right\} \text{or} \left\{\begin{array}{l} S \textit{ is either } P \textit{ or } Q \\ S \textit{ is either } \overline{Q} \textit{ or } R \\ \therefore S \textit{ is either } P \textit{ or } R \end{array}\right.$$

The following argument may serve as an illustration of this very rare type of argument. *A commodity is either produced on a large scale, or it is costly; And it is either in great demand, or it is not produced on a large scale; Therefore a commodity is either in great demand, or it is costly.*

This is the only type of pure alternative syllogism, abstracting from such minor variations as arise from differences in the quality of the alternatives, which (as already explained) may be either affirmative or negative.

§ 3. *Mixed Disjunctive Syllogisms.*

It is possible to draw a mediate inference from an alternative major premise and a categorical minor premise. After what has already been said about the meaning and implications of the alternative type of proposition, it should be obvious that the only kind of case in which a mediate inference can be drawn from an alternative major premise and a categorical minor premise is when the minor premise denies an alternative of the major premise. Thus:

$$\left.\begin{array}{l} \textit{Either } A_1 \textit{ or } A_2 \\ \textit{Not } A_1 \\ \therefore A_2 \end{array}\right\} \text{or} \left\{\begin{array}{l} S \textit{ is either } P \textit{ or } Q \\ S \textit{ is not } Q \\ \therefore S \textit{ is } P \end{array}\right.$$

This is the only type of mixed disjunctive syllogism, allowing for the fact that there is no significance in the order of the alternatives of the major premise, and that the alternatives may be negative as well as affirmative, and that consequently the minor premise and the conclusion may be affirmative as well as negative. The following arguments may serve as illustrations of the mixed disjunctive syllogism.

(1) *The light by which we see the moon, when it is beyond the reach of the direct rays of the sun, is due either to the moon's own light or to earth-light* (that is, light reflected from the earth) ; *But it is not the moon's own light* (or the moon is not self-luminous) ;

∴ *It is due to earth-light.*

(2) *Heat is either a kind of substance* (stuff) *or a kind of energy ;*
But it is not a substance ;
∴ *It is a kind of energy.*

CHAPTER XIV

DILEMMAS

§ 1. *Principal Types of Dilemma.*

A *dilemma* is an hypothetico-disjunctive syllogism, or a mediate argument, based on an hypothetical major premise and an alternative minor premise.[1] Now a little reflection will show that with a single hypothetical major premise it is impossible to employ a disjunctive minor premise, because a single hypothetical proposition presents no opportunity for an alternative assertion. With a single hypothetical proposition as major premise, the minor premise, whether it posits the antecedent or denies the consequent, is categorical, not alternative, and the whole argument is consequently a mixed hypothetical syllogism, not a dilemma. It is only when the major premise consists of two hypothetical propositions that the minor premise can be alternative, and either affirm alternatively the two antecedents, or deny alternatively the two consequents. The dilemma, like the mixed hypothetical syllogism, has two principal forms, the *constructive*, and the *destructive*. If the disjunctive

[1] Hence the dilemma is sometimes included among both mixed hypothetical and mixed disjunctive syllogisms. But to avoid ambiguity it is best to confine the terms *mixed hypothetical* and *mixed disjunctive syllogisms* to those cases in which the minor premise is a categorical proposition.

minor premise posits (alternatively) the two antecedents of the hypothetical major premise, then the dilemma is called constructive; if the minor premise denies (alternatively) the two consequents of the hypothetical major premise, then the dilemma is called destructive. The following are the principal symbolic forms of the dilemma.

(1) *Complex Constructive Dilemma*

If A_1, *then* C_1, *and if* A_2, *then* C_2
Either A_1 *or* A_2
$\therefore$ *Either* C_1 *or* C_2

(2) *Complex Destructive Dilemma*

If A_1, *then* C_1, *and if* A_2, *then* C_2
Either not C_1 *or not* C_2
$\therefore$ *Either not* A_1 *or not* A_2

The following arguments may serve as illustrations of the two types of dilemma respectively:

(1) *If heat is a stuff its quantity must vary with the volume of the substance which contains it, and if heat is a kind of energy then its quantity must vary with the amount of energy expended;*
Heat is either a kind of stuff or a kind of energy;
$\therefore$ *The quantity of heat must vary either with the volume of the containing substance, or with the amount of energy expended.*

(2) *If an examiner is tender-hearted he will pass weak candidates, and if he is just he will reject them;*
But he must either reject them or pass them;
Therefore an examiner is either not tender-hearted or not just.

It is usual also to distinguish between *simple* dilemmas and *complex* dilemmas. The foregoing schemas and arguments are illustrations of complex dilemmas, because the major premise in each case has two distinct antecedents and two distinct consequents. When the major premise has the same consequent for both antecedents, or the same antecedent for both consequents, then the dilemma is called simple. If there are two antecedents and only one consequent the simple dilemma can be constructive, but not destructive, because there are not two consequents to be denied alternatively in the minor premise, and the resulting argument, if destructive, cannot be a dilemma, though it can be a valid mixed hypothetical syllogism. When there are two consequents and only one antecedent in the major premise the resulting simple dilemma can be destructive, but not constructive, because there are not two antecedents to be posited alternatively in the minor premise, and so the resulting argument, if constructive, cannot be a dilemma, though it may be a valid mixed hypothetical syllogism. The principal forms of the simple dilemma are the following:

(3) *Simple Constructive Dilemma*

If A_1 *or* A_2, *then* C
Either A_1 *or* A_2
$\therefore$ *C.*

(4) *Simple Destructive Dilemma*

If A, *then both* C_1 *and* C_2
Either not C_1 *or not* C_2
$\therefore$ *Not* A.

The following arguments may serve by way of illustrations of the two types of simple dilemma :

(3) *If the miners have to work longer or to earn less, they will be dissatisfied ;*
But they must accept either longer hours or reduced wages ;
Therefore the miners will be dissatisfied.

(4) *If the coal industry were in a sound condition the miners and the mine-owners would be contented ;*
But either the miners or the mine-owners are discontented ;
Therefore the coal industry is not in a sound condition.

§ 2. *Difficulties and Faults of Dilemmas.*

The dilemma is a difficult form of argument merely because it happens so rarely that a problem, or situation, can be expressed exhaustively, or even adequately, in two alternatives ; and when the alternatives are not exhaustive, then the conclusion is not valid. At the same time it must not be supposed that there is anything inherently wrong with the dilemma. The way in which dilemmas are sometimes used, or rather abused, for rhetorical purposes, or in jest, is rather apt to convey the impression that the dilemma is a kind of sophistical trick rather than a sound form of argument. But such a view is quite erroneous. When the alternatives expressed in the minor premise of the dilemma are exhaustive, and the terms employed are not ambiguous and misleading, the dilemma is quite sound. Nearly always, if not absolutely always,

when a dilemma is invalid it is due to the fact that the minor premise does not exhaust all the possibilities, so that there remain possibilities leading to other results than those stated in the conclusion. And the defect is often concealed by ambiguous language which gives the minor premise the appearance of exhausting the alternatives when it really does not. Take, for example, the kind of arguments one sometimes finds in reports on inquests in connection with so-called peculiar people. The excuse for not calling in medical help usually runs somewhat as follows:

If the deceased was destined to recover, then medical aid was unnecessary, and if he was not destined to recover, then medical aid was futile;

But he was either destined to recover or not destined to recover;

Therefore medical aid was either unnecessary or it was futile. (In other words, *there was no point in getting medical help*.)

Here the dilemma looks sound, and yet most people feel that it is not. It looks sound because the alternatives stated in the minor premise appear to be exhaustive, for they look like a pair of contradictory alternatives (*destined* . . . and *not destined* . . .), and contradictory terms are collectively exhaustive. As a matter of fact, however, the alternatives are not exhaustive, in fact the most important possibility is omitted altogether, and its omission is veiled by a mere ambiguity. The ambiguity is this. In the major premise the second hypothetical proposition is only plausible because we assume that "not destined to recover" means "destined not to recover"; in the

minor premise "not destined to recover" only looks like the contradictory of "destined to recover" (and therefore as its completing or complementary alternative) because it is taken in a much wider sense than "destined not to recover," as including, in fact, not only this alternative, but also the case of there being no destiny at all. Now, obviously, if there is no such thing as fatalistic destiny, then everything might depend on the medical help called in in case of illness. This is the common-sense view, yet this possibility is entirely omitted from the argument—to say nothing of the possibility of a conditional destiny depending on certain measures being taken. The real possibilities may be indicated in the following scheme. A man's lot may be either (*a*) *destined* or (*b*) *not destined*; and if (*a*) *destined*, then the destiny may be either (*i*) conditional on certain steps being taken, or (*ii*) unconditional. The above dilemma as a matter of fact is tacitly based on (*a*) (*ii*) alone, and quietly ignores (*a*)(*i*) and (*b*) altogether.

§ 3. *The So-called Rebuttal of False Dilemmas.*

The commonest type of dilemma is the complex constructive, and as dilemmas are frequently invalid there has come into vogue a special device for refuting the complex constructive dilemma when it is suspected of inaccuracy. The device is known as that of *rebutting* a dilemma, and consists in transposing the consequents of the major premise, changing their quality, and then proceeding as usual. Let the original dilemma have the form:

$$\textit{If } A_1, \textit{ then } C_1, \textit{ and if } A_2, \textit{ then } C_2$$
$$\textit{Either } A_1 \textit{ or } A_2$$
$$\therefore \textit{Either } C_1 \textit{ or } C_2$$

Then the rebuttal will assume the following form :

If A_1, *then not* C_2, *and if* A_2, *then not* C_1
Either A_1 *or* A_2
$\therefore$ *Either not* C_2 *or not* C_1

For example, let the original dilemma be the one given on page 125, then its rebuttal will read as follows :

If the deceased was destined to recover, medical aid was not futile, and if he was not destined to recover, medical aid was not unnecessary ;
But he was either destined to recover or not destined to recover ;
Therefore medical aid was either not futile or not unnecessary. (In other words, *there would have been no harm in calling in medical aid.*)

For some reason or other the process of rebuttal has almost escaped criticism. It is possible that with an uncritical audience a so-called rebuttal may produce a favourable impression. As a rhetorical trick the device may, therefore, have some value. But logically it is worthless. Any complex constructive dilemma, even the soundest, can be "rebutted," and this alone should have made logicians suspicious of its value. The fact is that the "rebuttal" does *not* rebut the original conclusion at all. This should be obvious from a comparison of the original conclusion with the alleged refutation. The original conclusion is *Either* C_1 *or* C_2, and the alleged refutation is *Either not* C_2 *or not* C_1 ; but they are perfectly consistent with each other.

As has already been explained, the usual fault of an unsatisfactory dilemma is that the alternatives stated

in the minor premise are not exhaustive. The most effective way of really rebutting a dilemma is to point out what possibilities have been overlooked, and to show up such ambiguities as may lurk in the argument.

§ 4. *Abridged and Concatenated Disjunctive Syllogisms.*

It only remains to point out that the account given in Chapter XI of abridged categorical syllogisms and chains of categorical syllogisms applies, *mutatis mutandis*, also to disjunctive syllogisms, pure and mixed, and to dilemmas.

CHAPTER XV

INDUCTIVE AND CIRCUMSTANTIAL INFERENCE

§ 1. *Inductive Inference.*

The inferences discussed in the preceding chapters were all such as could be drawn from given premises, or could be evaluated in the light of given premises, provided one had sufficient knowledge of the language in which they are expressed to understand the meaning and implication of the main types of propositions, singly or in certain combinations. This does not mean that all these discussions turned on purely verbal matters. Far from it, for propositions express judgments, or thoughts, and thoughts are concerned with reality. But, for reasons already explained, we assumed that the propositions constituting the premises were there somehow, and that we were only concerned with the unfolding of their implications, not with the problem of their origin or derivation. If, however, it be asked now how the requisite propositions are obtained, then the answer is that some are obtained by direct observation, some by intuition, some by inference from other propositions, and some by inference from facts of observation. Singular propositions and particular propositions are commonly the result of direct observation, or of inference from other propositions so obtained. But general proposi-

tions, though many of them are inferences from other general propositions, are in the last resort obtained as inferences, or generalizations, from observations, and are not the mere equivalents of observations, in the way in which many singular and particular propositions may be said to be. Moreover, there are even singular and particular propositions which are likewise not the bare equivalents of observations nor inferences from other propositions, but inferences from observations.

Now the methods by which general propositions, or generalizations, and certain particular propositions like those expressing certain statistical regularities, are obtained from observed data, are known as the methods of science, and constitute the main problem of what is sometimes called *Methodology*, sometimes *Scientific Method*, and sometimes *Inductive Logic.* They are dealt with fully in *The Essentials of Scientific Method*, the companion volume to this, and need not be described or discussed here. One or two points, however, may be noted.

The methods of science can, of course, be described in general terms, and, when so described, can be shown to involve certain of the types of inference described in the preceding chapters. But the successful applications of the scientific methods involves a great deal more than that. It involves a familiarity with the facts investigated, and an insight into them that cannot be usefully formulated at all, but which may prompt conclusions which are felt to be sound in spite of certain shortcomings when gauged in the light of the rules of purely formal inference.

Again, when such generalizations have been made and formulated in clear propositions, these proposi-

tions can be made the starting-point of immediate and mediate inferences which may help one to explain present and past events, or to anticipate future events. But the transition from general proposition to events, though by no means so easy as to be fool-proof, is much easier than the transition from observed events to trustworthy generalizations. For one medical man who makes a new discovery there are probably more than ten thousand who do not, but who can safely and usefully apply old generalizations to new cases.

Again, there are spheres of thought where the question of inferring general propositions from observed cases scarcely arises, but where there is abundant scope for applying all the rules that concern valid inference from given propositions. Law is a case in point. The numerous laws in existence are, in one sense, so many general propositions which have to be accepted as authoritative, and used as premises of all kinds of syllogistic inferences. The work of inferring general propositions (or laws in the scientific sense) from observed data holds no important place in the business of law-courts and of lawyers. To some extent, therefore, the study of what may be called inference from propositions can be carried on by itself, independently of the problems of drawing general inferences direct from the facts. Some logicians, in fact, regard such inferences from propositions as the real province of Logic proper, or *Pure Logic*, as it is sometimes called.

Of course, a knowledge of the kind of inferences dealt with in the preceding chapters should be helpful in connection with the application of scientific methods. For, after all, what is aimed at in scientific investigation may be described, from one point of view, as the discovery of propositions from which the observed

facts might have been inferred—the inferribility of the facts from such propositions constituting usually the explanation of the facts. That is why *induction* is commonly described as the *reverse of deduction.* But the main point is to realize that the study of scientific method is inevitably less abstract than the study of inferences from propositions, and is consequently on a rather different footing.

There is one kind of inference which, although it is not concerned with generalization, is like inductive inference inasmuch as it is inference from facts rather than from propositions, is a reverse process, and proceeds by way of hypothesis. But it is a very common type of inference, and is not usually included among the methods of science, so that something might appropriately be said about it here. It is known as inference from circumstantial evidence.

§ 2. *Inference from Circumstantial Evidence.*

Circumstantial evidence is particularly common in connection with attempts to trace criminals, but of course it is not confined to such cases. What is meant by *circumstantial evidence*? The term is commonly used by way of contrast with what may be described as evidence bearing directly on the main problem, and denotes evidence concerning, or consisting of, certain facts or factors which, so to say, *surround* the main event. For example, take the case of a theft. If the thief is caught in the act, that is direct evidence of the most conclusive kind. Usually, however, people who plan a crime will take every precaution they can think of against red-handed discovery, in fact, against any kind of direct observation of their crime. Such direct evidence is, accordingly, un-

common. Frequently, however, there are "circumstances," that is occurrences surrounding the principal act, and more or less connected with it, which betray the criminal. It is these "circumstances" that constitute the *circumstantial evidence*. What usually happens is this. Certain objects or occurrences, observed in the neighbourhood of the locality where the crime was perpetrated, are felt to be connected with the crime or the criminal. These are traced as far as possible, and linked up with other facts and occurrences until they suggest an hypothesis about the identity of the criminal. The hypothesis, if satisfactory, will be such that it links up the otherwise disconnected objects and occurrences into one connected whole or system. And, in the absence of serious counter-evidence, the hypothesis which gives the most consistent and complete picture of the principal event in question and its circumstances will be accepted as the true account. It is important, of course, that no circumstances unfavourable to the hypothesis should be ignored. An arbitrary selection of circumstances may send innocent people to prison or even to the gallows. Such things have happened.

Perhaps the simplest illustration of inference from circumstantial evidence is the process of piecing together the parts of a picture puzzle. Here we have a number of separate parts which are supposed to be connected somehow, or to belong together, and the solution of the puzzle takes the form of putting them together in such a way that they present a consistent and harmonious whole. Of course the problems of practical life are not often solved so completely as picture puzzles, or cross-word problems, which leave no parts over that cannot be accounted for.

Inference from circumstantial evidence is a very common method of historical reconstruction, even apart from the unravelling of political crimes. The task of the political historian may be described as that of so utilizing the available evidence as to reconstruct by means of it the drama of the events in question. It is in some ways like piecing together the pieces of a puzzle picture, but of a puzzle picture of which many parts are missing, while others have been damaged, or distorted, or faked. The result is that there is far more scope for the play of individual fancy in the case of historical reconstruction than there is in the assembling of the parts of an ordinary picture puzzle. What has just been said of political history holds good likewise of literary history, and of all kinds of historical and biographical researches.

Inference from circumstantial evidence can be abundantly illustrated from the files of newspapers, and from political and literary histories. Any good account of, say, the chronology, etc., of the plays of Shakespeare (to say nothing of the Bacon-Shakespeare controversy) will afford examples of this kind of inference. But the following quaint example, culled from the recent speculations of a contemporary archæologist, may serve our purpose.

Everybody is familiar with the expression "the island of the dead." Is the expression merely a poetic fancy, or does it express an early belief in the actual existence of such an island; and, if so, which island did it refer to? If we could cross-examine the early writers who used the expression they might tell us what they really believed. That would have been *direct* evidence. But that is not available. One has, consequently, to rely on "circumstantial evidence,"

that is, such allusions to "the island of the dead" as can still be read in old writings.

Now the principal relevant "circumstances" may be summarized as follows. According to Greek mythology the underworld of the dead is a land of darkness, where the sun never breaks through the mists and clouds. According to Homer, the land of the Cimmerii formed the entrance to the underworld, and this land was situated "at the end of the deep Ocean." The Styx, which the dead had to cross in boats, is also described as "an arm of the Ocean." And in the passage in the Odyssey, which describes the journey of the slain wooers across the Styx to the underworld, there is a reference to some "white cliffs" which they passed on their way. Now Pluto was not only the god of the underworld, but also of metal ores. The Cornish coast of Britain was well known to Phœnician mariners, who came there for tin. This coast also marked the limit of maritime adventures in those days ("the end of the deep Ocean"). To the sunburnt Mediterranean the clouds and mists and pale complexions he encountered in Britain may have seemed strange indeed. Again, the Celts usually went by the name of *Cymri*, not very unlike Homer's *Cimmerii*. Lastly, in the fifth century A.D. Procopius relates explicitly a legend according to which the souls of the dead sail in ships from the Gallic coast to an island called *Britia*, which he assumed lay somewhere north of Scotland, and was inhabited by Britons, Saxons and Frisians.

Such are the data, or "circumstances." The hypothesis suggested is that originally Britain was really regarded as the island of the dead. By the fifth century A.D., however, Britain was sufficiently well

known to render this identification impossible, and so Procopius located the "island of the dead" farther north. Whether the hypothesis is true or not need not be discussed here. The main point is that the hypothesis gives meaning to a number of things which would otherwise appear to have no significance.

CHAPTER XVI

SOME GENERAL PROBLEMS OF INFERENCE

THERE are certain general problems in connection with inference which may be considered briefly in this chapter.

§ 1. *The Objective Basis of Inference.*

The acts by which we draw inferences are obviously mental acts. Suppose for a moment that there were no minds capable of thinking, in that case there could be no such thing as inference. But this does not mean that inference is entirely a subjective matter, entirely the product of consciousness. Inference has an objective basis, without which it would be no more significant than the capricious play of fancy. The objective basis of inference is the actual connection between things, or events, in the world of reality. A typical kind of objective connection between events is the causal connection. And in that case it is obvious that the causes of events, when known, become the reasons for our inferences. Take away such objective causal connections, and human reasoning becomes but an idle causerie, or at most an unaccountable habit of associating ideas. Valid inference, consequently, has a two-fold basis. It presumes objective connections in the world of reality; and it assumes the possibility of our somehow apprehending these objective connections so as to use them as grounds for inferences.

This at once raises the question of the relation between the objective connections between the events in the world of reality to which our thought refers, and the logical connections between our thoughts when we reason about those events. The usual tendency to simplify things generally expresses itself in this instance in an attempt to reduce either to the other, and to recognize one kind of nexus only. On the one hand there are the so-called idealists who endeavour to reduce the connections between material events to logical connections, and so come to regard the universe as essentially a structure of ideas (or Ideas, with a capital). On the other hand there are the materialists who endeavour to reduce the logical connections between ideas to causal connections between cerebral processes. Philosophers commonly support the former view, while men of science (if they think about these matters at all) usually favour the latter view. There are numerous exceptions, of course, in the ranks of both philosophers and scientists. There is really very little to be said in favour of either of these extreme views; and the whole problem has no special relevance for the study of Logic as such. The problem pertains to the Theory of Knowledge, or to Metaphysics generally. So far as Logic is concerned, there is no point in attempting to merge the material nexus in the logical, or the logical in the material. The two can be regarded simply as co-existent or parallel and apparently different.

Even so it remains true that the systematic character of our knowledge is based on the systematic character of reality. The more adequately we endeavour to understand anything, the farther must we pursue its ramifications in the whole system of reality, and

in the end (or ideal limit) the complete knowledge of anything (even of the little "flower in the crannied wall," on which Tennyson mused) would require a knowledge of the entire system of things, which alone, according to Spinoza, is *substance*, that is, self-supporting, both objectively and logically.

§ 2. *Inference and the Particular.*

According to some, inference is always from particular (or individual) cases to other particular cases, and when a general proposition is employed in deductive inference, the general proposition serves only as a memorandum of individual cases observed, and the inference is really from those individual cases, and not from the general proposition. On the other hand most logicians go to the other extreme and maintain that inference is never from particulars, but always from some general or universal character or aspect of the particulars in question.

The controversy is unfortunately complicated by various philosophical considerations which are of little, if any, logical importance. The main points of interest may be set out as follows.

If by "particular" is meant anything apprehended in such a way as to involve no reference to anything general (in other words, in such a way as to involve no comparison whatever with anything else, no apprehension of any of its characteristics, or features, or qualities as common to it and other things), then certainly there is nothing that we think or speak about that can be described as particular. The apprehension of such particulars, if and when it takes place, is inevitably speechless, inarticulate. At that stage there can be no such thing as the apprehension

of an object, or event, and its place in any kind of objective order. At that stage, therefore, there can be no such thing as logical inference, or thought of any kind. There can only be some kind of instinctive reaction, or practical orientation, no more. In fact, at that stage, the possibility of forming judgments, even the simplest judgments, has not yet emerged. For, as has already been explained in Chapter II, § 4, even the most rudimentary judgment requires at least one concept for predicate; and a concept is never particular in the sense now supposed. In this sense, therefore, it may be said that inference, or indeed, thought generally, is not concerned with the particular as such. That, however, is not really what is usually meant when we speak about the "particular."

If, on the other hand, the term "particular" is used in an unsophisticated sense, as synonymous with "individual" or "singular," that is, as denoting this or that member of a class or kind, then it should be obvious from the foregoing chapters that some inferences are from particulars to particulars, other inferences are from particulars to general propositions, others are from general propositions to a particular case or cases, and yet others are from general propositions to general propositions. Thus, for instance, the mediate inferences discussed in Chapter VIII, and inference from circumstantial evidence, discussed in Chapter XV, are certainly inferences from particulars (in the sense of singular propositions, or individual cases), and so are most inductive inferences. On the other hand, deductive inference proper is inference from general propositions, so that neither of the extreme views (usually associated with the name of Mill and his opponents) is really

correct. Inferences are not always from particulars nor are they always from the general, but they are sometimes from the one and sometimes from the other, and sometimes from both.

What is really characteristic of all reasoning is its exploitation of connections. This characteristic is perhaps most obvious in the case of inference from circumstantial evidence, where we see very clearly, even in comparatively simple cases, the feature of interconnection between the circumstances involved, and the corresponding convergence and interlinking of the evidence which leads to the inferential construction of the complex, systematic whole. In the simpler types of inference, sometimes described as "linear" inference, the interconnection is not so marked. But the difference is only one of degree. As already remarked earlier in the book, the farther a problem or a discussion is pursued, the more ramified, or systematic, does it become. Logicians who are committed to the extreme view that inference always involves something general, or what they call a "universal," will probably insist that even inference from circumstantial evidence involves a universal, only in this case it is what they call a "concrete universal," as opposed to an "abstract universal." But to call a system a "universal" is to attempt to save a theory by obscuring the real issue.

§ 3. *The Principle of Uniformity of Reasons.*

There is a certain fundamental assumption of all inference which appears to have escaped attention except in so far as to mislead some logicians in their conception of the ultimate type of all inference. The assumption, or postulate, in question may be formulated

as follows: *Whatever is regarded as a sufficient reason in any one case must be regarded as a sufficient reason in all cases of the same type.* Or, to express it negatively, *Nothing can be regarded as a sufficient reason in any one case unless it can also be regarded as a sufficient reason in all cases of that kind.* The principle may be regarded as the logical parallel to the Principle of the Uniformity of Nature but is more comprehensive. Phenomena of the same kind exhibit the same kind of objective connections. Conclusions of the same kind must be explained in the same kind of way. At least that is so potentially, for in some cases (for instance in most cases of inference from circumstantial evidence) we are concerned with what is unique, or not likely to happen again. In any case, the principle just formulated is the counterpart or complement to the principle which lies at the basis of the warning against the fallacy known as *argumentum a dicto simpliciter ad dictum secundum quid,* and its converse. The objection to this kind of argument is that it involves a neglect of the sound maxim that "circumstances alter cases." What is true of cases of a certain general type may not be true of cases in which special circumstances are operative. Now the converse or the complementary principle to it is this: Similar cases must be treated in the same way, unless it can be shown that there are special circumstances requiring special consideration.[1] And that is virtually the Principle of the Uniformity of Reasons formulated above.

Now this principle of the uniformity of reasons is at the back of all reasoning. And every attempt made

[1] A glimmer of this seems to be visible in some uses of the proverb "What is sauce for goose is sauce for gander."

to formulate the general principle of any special type of inference is made out of deference to the principle of the uniformity of reasons. Examples of such principles we may find in the *dictum de omni et nullo* (or its substitutes) in connection with syllogisms of Fig. I, or the axiom that "things which are equal to the same thing are equal to one another," and so on. Now such general principles are really assumptions at the back of the corresponding types of argument; they are not actual premises of the arguments themselves. Failing to grasp this difference between an actual premise and a postulate, or principle, some logicians have endeavoured to show that all arguments, including inductive arguments, etc., are really syllogistic arguments, and no more. This they can only do by treating the relevant postulate as the major premise, and the whole of the actual premises as minor premise. But if this is carried through consistently then every ordinary syllogism would really be a double syllogism—one as it actually is, and another when the general principle of syllogistic inference is treated as the major premise, and the actual premises are made to function together as minor premise. And even then the principle would still be assumed. The whole attempt is extravagant. Even if much more could be said in its favour than is the case, the student of actual inference would still find it far more important and profitable to study the *differences* between the main types of inference than their alleged sameness.

§ 4. *Concluding Remarks.*

The study of Logic has sometimes been described as a kind of refined intellectual game carried out accord-

ing to certain rules. Its value as a mental discipline might have been very considerable even so. In reality, however, it is as serious a thing as life itself. Pursued in the right spirit, the study of Logic and Scientific Method is an invaluable training in the art of self-criticism, which is so necessary to the peace and welfare of humanity. Some of the world's profoundest thinkers looked to the cultivation of Reason as the foundation of the future unity and harmony of mankind. This touching faith in the power of Reason may be unseasonable in an age notorious for its cult of the Irrational. But, if the Principle of the Uniformity of Reasons is properly understood, the justification of this faith will become clear, and so will Spinoza's view that "it is the passions that divide men, Reason brings them together." For it is in accordance with this Principle that (as Spinoza says) "men who seek their own welfare under the guidance of Reason desire nothing for themselves which they do not wish also for the rest of mankind." And it is the same principle which is at the basis of Kant's formulation of the moral law: "Act in such a way that you can will the maxim of your act to become a universal law."

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